ISSN 2045-5119

University of Oxford
Department of International Development

TMCD Working Paper Series
No. 084

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Modeling Uncertainties and Gender Differences in Entrepreneurial Decision Making

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Draft of March 26, 2023, this version October 25, 2023.

Abstract: This paper addresses the unexplained phenomenon of gender differences in entrepreneurial decision-making, particularly in business sectors involving innovation and risk. Although the topics of risk attitudes and uncertainty have been studied comprehensively from a gender perspective, research that combines different forms of uncertainty and links them with women's entrepreneurial decision-making has not been done to the best of our knowledge. This paper provides a theoretical framework involving models on risk, ambiguity, perceptions of risk as in cumulative prospect theory, and asymmetric information. It unifies and connects these models through specific assumptions on utility functions in the presence of uncertainty and uses bias discount factors to denote how strongly biases devalue outcomes. Conclusions from the models indicate that a range of external factors—such as race and access to education and social networks—may interact with the internal behavioral biases of an entrepreneur, such as overconfidence, risk appetite, altruism, and trust in others. By detailing and analyzing theoretical gender-type differences and their effects on entrepreneurial decision-making, we find that traditional one-dimensional gender policies are not sufficient to address complex gender issues. Rather, we suggest a more multi-dimensional and holistic policy approach that combines traditional gender policies and policies for long-term sustainability, encouraging firms to be more socially and environmentally conscious, and demanding less "greedy works" as Nobel laureate Professor Goldin also argued. It aims to create a new business eco-system that is more feminine-friendly and balanced, considering gender-type differences and incentive biases to address the current worldwide gender disparities in entrepreneurship.

Keywords: Decision making under Uncertainty; Entrepreneurship; Gender

JEL-Codes: D01; D81; D91; L26; L38; J16; Q01

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[2] The authors would like to thank Professor Xiaolan Fu (Founding Director of the Technology and Management Centre for Development at the University of Oxford), Professor Elettra Agliardi (Department of Economics at the University of Bologna), and Professor Clapperton Chakanetsa Mavhunga (Science, Technology, and Society at Massachusetts Institute of Technology [MIT]) for their support and for offering detailed comments on the initial drafts of this research. Any mistake or omission in this study is the sole responsibility of the authors.

1. Introduction

A pressing issue worldwide is the gender gap in innovation entrepreneurship and the factors that contribute to its existence (Global Entrepreneurship Monitor, 2018; Shinnar et al., 2018); even in societies like the USA and the UK with strong gender equality, we still find serious gender disparities in entrepreneurship in areas like artificial intelligence (Farah, 2023). There are many economic benefits of increasing women's participation in entrepreneurship such as creating employment opportunities, reducing poverty, and increasing support for family income. It also enhances the economy's growth in productivity and sustainability, which are crucial in creating a more affluent society in today's competitive and globalized world (Akehurst et al., 2012; World Bank, 2012; Sajjad et al., 2020). According to the report of the McKinsey Global Institute (2015), by making women equal partners with men, an estimated USD 12 trillion or more could be added to the world's annual GDP by 2025. This increase in GDP would be due to the female half of the population being put in a position to develop their full potential of knowledge and ability to become entrepreneurs, fostering innovation and productivity, which increases overall well-being (Love et al., 2023).

The gender gap in entrepreneurship has gradually been closing over the past twenty years, increasing the number of women entrepreneurs; never-the-less, there still are fewer women than men pursuing entrepreneurship. And those who do are often in the less capital-intensive sectors including personal services (European Commission, 2016; Priyadarshini and Ramakrishnan, 2016; Cardella et al., 2020). Moreover, the COVID-19 pandemic has greatly reduced progress in bridging the entrepreneurial gender gap (OECD, 2021).

The topic of gender-type differences in entrepreneurship — i.e. characteristics typically associated with and expected of women or men — and recommendations to handle their effects is largely covered in the existing literature. It is useful to differentiate between gender-type and gender because the expected, typical characteristics of men and women do not define every man and woman's actual roles and behaviors. We start by assuming there is a female population and a male population: the females have a large fraction of female-gender-type people and a small fraction of male-gender-type people, whereas the males have a large fraction of male-gender-type people and a small fraction of female-gender-type people. Thus, we account for the literature showing mixed results for definitive gender differences.

Our research aligns well with the research agenda of Professor Claudia Goldin, the 2023 laureate for Nobel Prize for Economics, both confirming important results and — in the context of gender and entrepreneurship — addressing questions not yet answered. Smith (2023) states: "[Professor Goldin's] research illustrates how the process of closing the wage gap has been uneven throughout the course of history, changing in line with social norms and expectations about their career prospects and roles at home". Professor Goldin argues that inequalities at home can sustain inequalities at the workplace, which is confirmed in our model of the choice of leadership in a family firm in Section 3.3.

Regarding 'greedy jobs', i.e. highly paid, stressful jobs which require work to be prioritize over all else, Smith (2023) quotes Professor Goldin saying: "For example, why do women and not men step back from these higher paid opportunities? […] and how can we make these 'greedy jobs' less demanding, without making them less productive? That's the next frontier we have to explore." Our research supports the critique by Goldin (2023) of existing policy measures, who said "These responses haven't worked to erase the differences in the gender pay gap. And they will never provide a complete solution to gender inequality, because they treat only the symptoms." For the context of gender and entrepreneurship, our policy recommendations address exactly this question to seek complete and balanced solutions to treat the 'root' of the symptoms. The answer we arrive at is that traditional one-dimensional gender policies are not sufficient to address complex gender issues. Rather, we suggest a more multi-dimensional and holistic policy approach is required, which combines traditional gender policies with policies for long-term sustainability, encouraging firms to be more socially and environmentally conscious. It aims to create a new business eco-system that is more feminine-friendly and balanced, considering gender-type differences and incentive biases to address the current worldwide gender disparities in entrepreneurship.

This paper is inspired by Sent and Staveren's (2019) survey paper which finds that results on gender differences are typically mixed, meaning that the extent to which there are differences can vary depending on the context. Specifically, these are differences in various forms of uncertainty such as risk attitudes, loss aversion, ambiguity, confidence, degrees of optimism, and more (e.g. Adachi and Hisada, 2017; Bjuggren and Elert, 2019; Borghans et al., 2009; Brebner, 2003; Chen et al., 2018; Hügelschäfer and Achtziger, 2014; Prasad et al., 2021). There is a general observation, though, that female-type persons are more risk-averse, emotional, and altruistic, but less confident and trusting in others, than male-type persons (Barett et al., 1998; Harris and Jenkins, 2006; Hügelschäfer and Achtziger, 2014).

Although a significant amount of literature focuses on such topics, a more comprehensive theoretical framework for modeling uncertainty and gender-type differences in entrepreneurial decision-making could put some of the results in perspective and may provide additional insights. Therefore, this paper provides a broader framework to address the key question of our paper: "Why are there fewer female entrepreneurs, especially in sectors benefiting from risk-seeking for innovation?" "What causes the gender-type differences in entrepreneurial decision-making, and what avenues can increase the number of female entrepreneurs in innovation?" "Are these causes rooted in collective, communal decision-making, or in individual, agentic decision-making?". Essentially, we ask: "What kind of business setting do we want to achieve?" "And if we aim to leave no one behind in positive change, how is the gender dimension currently missing?" "What are the policy options for addressing this?"

Expected utility, ambiguity theory, prospect theory, and asymmetric information are approaches used in economics that have relevance for understanding real-life gender differences in entrepreneurial decision-making, and their multiplicative effects on inequality. Thus, we follow Knight (1921) and take the perspective of a decision-maker choosing between pursuing entrepreneurship or not. Their decision will depend on their gender type, which relates to different characteristics that crucially affect the outcomes under these theories. In doing so, we analyze the effects of internal and external factors and establish a theoretical framework to address the unexplained phenomena.

There are internal factors and external factors affecting entrepreneurial decision-making. They are interconnected and reinforce each other but should be distinguished. Internal factors include altruism, trust, risk appetite, overconfidence, utility functions and indices, reference points, pessimism/optimism, and individual intertemporal discount factors reflecting patience. External factors include a systemic market bias discount factor on women's output/value, capturing the negative environment in which they operate. They could reflect institutional biases for different genders, races, and cultures, especially in education and in access to social and professional networks, as well as a lack of legal protection acknowledging the economic value of their contribution. Such negative external factors may seriously limit and distort the available choices as well as increase the performance gap between male and female entrepreneurs (Oggero et al, 2020; Oladipo et al, 2023).

In general, we define internal factors as what is experienced in the decision-makers mind. They are characteristics of the decision-maker. External factors, on the other hand, are defined as the characteristics of the environment that the decision-maker operates in, for example of the market or of other relevant institutions.

Our appropriately simplified theoretical framework focuses on various forms of uncertainty: risk in expected utility (Savage, 1954; Anscombe and Aumann, 1963); ambiguity as in Choquet expected utility (Schmeidler, 1989; Mukerji, 1997; Chateaneuf et al., 2007); and the decision-makers individual perception of risky choices as in prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). The theoretical framework identifies weights applied in aggregating valuations of the different potential outcomes of a choice, as a unifying way to identify the differences in how each decision model portrays uncertainty (Wakker, 2010). Additionally, it uses the certainty equivalent as a benchmark to explore a counterpart of the risk premium in the form of an “uncertainty premium” and “bias discount factors” as simple measures of the fractional loss due to various biases. We also provide examples of how the effects of ambiguity may be misinterpreted to be the effects of cumulative prospect theory, and vice versa.

We limit this paper's definition and scope of uncertainty to facilitate the analysis. The type of calculable risk we use as the starting point for our analysis is assumed to have the potential to enrich society with innovations. But it is of the same nature as purchasing lottery tickets, placing bets, or pure gambling. Therefore, its scope extends to the risks associated with irrational gambles or financial speculations that have no positive impacts on advancing humanity, as these too are related to entrepreneurship, innovation, and our main questions. In the spirit of Knight (1921), the discussions in the paper consider the decision-makers to be entrepreneurs facing Knightian uncertainty in their choices to advance innovations.

We would like to emphasize that the economic agency in our approach not only has calculative and emotional motives but also affects the individual decision-maker’s *subjective* context in which decisions are made. This differs from many experimental settings, such as that of Borghans et al. (2009), in which factors like ambiguity aversion are tested *objectively*. In settings where the population sample is not affected by contextual factors — for example, educational opportunities, gendered institutions, and social settings relating to network resources — the differences in the economic behavior of men versus women cannot be fully explained (Sent and Staveren, 2019).

In summary, our research analyzes and provides a deeper understanding of how inequality takes shape through the multi-layered dimensions of women's innovative entrepreneurship choices in a negatively biased external environment. Only after clarifying the chain of causes and effects that lead to inequality can we try to break the vicious circle. The structure of this paper is as follows: in Section 2, we introduce the theoretical framework of decision-making under uncertainty and how it depicts communal and agentic behavioral attitudes in entrepreneurs. Gender bias discount factors and two theoretical adverse selection models are introduced in Section 3, the models relating gender and gender-types to entrepreneurship. In Section 4, the theoretical insights about risk, uncertainty, and misperceptions are related to a range of gender-type differences and interactions in entrepreneurial decision-making. The interconnections between some of the behavioral characteristics are also explored. Section 5 discusses the key implications of our findings for the gender gap in innovative entrepreneurship. Our final section addresses policy implications and provides concluding remarks.

2. The Theoretical Framework

The general structure of rational decision-making is: each decision-maker pursues his or her objectives by using the instruments available to them, subject to institutional and physical constraints they face, and to the anticipated reactions of the others. We examine decision-making under uncertainty since we consider Knightian innovative entrepreneurs who are specialists in dealing with uncertainty (Knight, 1921). Here, uncertainty is a lack of knowledge about the probability of a state of nature's future condition or what the future outcomes of these states may be (Keynes, 1937; Groot and Thurik, 2018). Knight's intuition is captured in Kelsey and Spanjers (2004) using the entrepreneurial sharing rule for partnerships with ambiguity.

This section aims to introduce and unpack the meaning of uncertainty by discussing the differences between risk, ambiguity, and misperception. We present the corresponding three approaches of expected utility, ambiguity, and cumulative prospect theory, in Appendix 1. We analyze how these three approaches are distinct but also combine to constitute a coherent framework for assessing the gender dimension of entrepreneurial decision-making. The impact of an external, biased environment, as in, for example, theories of asymmetric information, is represented through "bias discount factors", denoting the fractional devaluation caused by the bias.

We focus on behavioral attitudes toward uncertainty. These include agentic attitudes like overconfidence and risk appetite, but also communal behavioral attitudes such as trust and altruism. These different perspectives of uncertainty help us to understand the origins of gender differences in entrepreneurial decision-making. They also point to avenues that can break barriers and increase the number of women entrepreneurs in innovation.

2.1 Expected Utility

The Expected Utility approach is where a decision maker behaves as if they evaluate uncertain prospects and their expected utility values. This expected utility is based on the decision maker's beliefs and is obtained from the sum of the utility values of the potential outcomes multiplied by their respective (subjective) probabilities:

$$U(x_1,\ldots,x_S\,;\,\pi_1,\ldots,\pi_S) = \pi_1\,u(x_1) + \ldots + \pi_S\,u(x_S) = \mathbf{E\{u(x)\}} = EU(x, \pi)$$

Here U denotes the expected utility function representing the preference relation for the pairwise comparison of uncertain prospects (x, π).[3] For each state (or potential future scenario) $s = 1, \ldots, S$, we use x_s to denote the income (or wealth) that results when state s materializes, and π_s to denote the (subjective) probability with which the decision maker expects state s to occur. Finally, u denotes the von Neumann-Morgenstern utility index, which denotes the utility $u(x_s)$ resulting from an income x_s, if and when this income is obtained.

The Expected Utility approach doesn't assume that the decision-maker consciously uses the expected utility equation to evaluate uncertain prospects. Rather, the approach is based on underlying behavioral assumptions about the decision maker's pairwise comparisons of uncertain prospects, tailored around the Sure Thing Principle (Savage, 1954; Abdellaoui and Wakker, 2020) or the Independence Axiom

[3] For a comprehensive discussion of the concept of prospects see Wakker (2010, Chapter 1).

(Anscombe and Aumann, 1963). Under the seemingly plausible assumptions on the decision-maker's pairwise comparisons of uncertain prospects, it follows by the force of mathematical logic that these comparisons are ***as if*** they result from a suitably specified expected utility function. Thus, for the Expected Utility approach to hold, it is not assumed that the decision maker is numerate or that they have been exposed to mathematics or statistics.

The expected utility function reveals the decision-maker's attitude toward risk. This risk attitude is reflected in the von Neumann-Morgenstern utility index u. Assuming that a higher income results in a higher value of the u, the decision maker's risk attitude follows from the curvature of the von Neumann-Morgenstern utility index. For a strictly concave utility index (i.e. $u''(x_s) < 0$) the decision maker is risk averse; for a strictly convex utility index ($u'' > 0$) the decision maker is risk seeking; and for a linear utility index the decision maker is risk neutral ($u'' = 0$).

A different approach to represent the attitude of a decision-maker towards risk is by comparing risky prospects to risk-free income. The certainty equivalent of a risky prospect is the risk-free income which the decision-maker considers to be equally good. That is, it is the risk-free income for which the decision maker is indifferent between obtaining this risk-free income CEQ and obtaining the risky prospect $(x_1,\ldots, x_S; \pi_1,\ldots,\pi_S)$. That is $U(CEQ\ ; 1) = U(x_1,\ldots,x_S\ ; \pi_1,\ldots,\pi_S)$. Or, put differently: $u(CEQ) = \pi_1\, u(x_1) + \ldots + \pi_S\, u(x_S)$.

Based on the certainty equivalent of a risky prospect, the risk premium of the prospect is defined as the difference between the expected income of the prospect ($EI = \mathbf{E\{x\}} = \pi_1\, x_1 + \ldots + \pi_S\, x_S$) and its certainty equivalent CEQ, that is $RP = EI – CEQ$. Now a decision maker is risk averse regarding a risky prospect if its risk premium is strictly positive (i.e. $RP > 0$), risk-seeking concerning the risky prospect if its risk premium is strictly negative ($RP < 0$), and risk-neutral concerning the risky prospect if its risk premium is zero ($RP = 0$). Intuitively, the risk premium can be interpreted as the monetary value of the sleepless nights of the decision-maker worrying.

A related way to represent the decision maker's risk attitude is by use of a "risk discount factor". This risk discount factor denotes the rate at which the utility of the decision is "discounted" due to the presence of risk. For a risky prospect $(x_1,\ldots, x_S; \pi_1,\ldots,\pi_S)$ the decision maker's risk discount factor DF is obtained as $DF = CEQ\ /\ EI$. For a risk-averse decision maker, $DF < 1$ since $CEQ < EI$. Similarly, for a risk-seeking decision maker, $DF > 1$, and for a risk-neutral decision maker $DF = 1$.

The basic concepts of "certainty equivalent," "risk premium," and "risk discount factor" can easily be generalized to broader settings relating to decision-making under uncertainty. This will be done in some of the sections to follow to facilitate the discussion and comparison of the different forms and settings of uncertainty addressed in this paper.

The presence of gender biases in decision-making under uncertainty is well established (e.g. Eckel and Grossman, 2002, 2008). Some of these biases may relate to different parameters in the expected utility function. But there are different approaches to decision-making under uncertainty that may equally, or at times even better, explain the gender gap in innovative entrepreneurial behavior regarding phenomena like differences in risk attitude, overconfidence, trust, and altruism. In the following sections, we summarize a selection of such theories as the basis for our further discussions.

2.2 Ambiguity

When contrasting risk with ambiguity, Keynes (1937) describes ambiguity as a form of uncertainty where "there is no scientific basis on which to form any calculable probability whatsoever. We simply do not know". In practical situations of uncertainty, we would consider some ambiguity to be present whenever we do not know "within bounds." The presence of ambiguity gives rise to incalculable risk. The "risks" in the situation of uncertainty become incalculable because the probabilities and/or the outcomes are not known within bounds (see e.g. Knight, 1921; Mukerji 1997; Wakker 2011).

When making decisions in situations of uncertainty, one would want decision-making to be based on what is known; in this context, any amount of "simply not knowing" creates serious issues. To deal with decision-making in the presence of ambiguity, we need to find new parameters to describe (the bounds of) the ambiguity of a situation, as well as its potential impact on decision-makers. In this, Ellsberg (1961) leads the way in at least three different aspects.

Firstly, the Ellsberg-paradox makes it credible that even rational decision-makers who are familiar with the Expected Utility approach may make well-considered choices that violate Savage's (1954) Sure Thing Principle and that are not consistent with the expected utility theory.

Secondly, Ellsberg considers this to be the consequence of a "lack of confidence" in the probability distribution, represented mathematically by "not knowing within bounds" which probability distribution applies.

Finally, a decision model is provided that incorporates these effects by extending the expected utility function to incorporate an aversion to "not knowing." This is achieved by introducing the decision maker's level of confidence γ regarding their probability estimate π, and the extent of not knowing reflected in the set of plausible probability distributions Γ. In the overall evaluation of the situation, the valuation when "knowing" (with weight γ), i.e. the expected utility based on the probability estimate π, is combined with the decision maker's valuation within the realm of 'not knowing' (with weight $1 - \gamma$), being the worst expected utility valuation for any plausible probability distribution p in Γ. Thus, Γ can be interpreted to denote the extent of not knowing, whereas $1 - \gamma$ can be interpreted to denote the intensity of not knowing.

The resulting utility function proposed by Ellsberg (1961) is as follows:

$$U(x, \pi, \gamma, \Gamma) = \gamma\, EU(x, \pi) + (1\text{-}\gamma)\, min_{\,p \text{ in } \Gamma}\, EU(x, p).$$

Here x denotes for $s = 1, \ldots, S$ the state contingent incomes $x = (x_1,\ldots, x_S)$. The decision maker's (subjective) probability estimate for the states is denoted by $\pi = (\pi_1,\ldots,\pi_S)$ over states. Furthermore, γ with $0 \leq \gamma \leq 1$, denotes the decision maker's 'level of confidence' in their (subjective) probability estimate π, with $1 - \gamma$ their lack of confidence. Thus $\gamma = 1$ denotes full confidence and $\gamma = 0$ denotes no confidence at all. $EU(x, \pi)$ denotes the expected utility function for a von Neumann Morgenstern (vNM) utility index u as seen before, and $EU(x, p)$ similarly for the probability distribution p. For given state contingent incomes x, the expression $min_{p \text{ in } \Gamma}\, EU(x, p)$ denotes the minimum expected utility value obtained for $EU(x, p)$ over all "plausible" probability distributions p in Γ.

Convincing as Ellsberg (1961) is, the proposed utility function is not compatible with the expected utility function except for the case of "full confidence," i.e. for $\gamma = 1$. So, for $0 < \gamma < 1$, the seemingly plausible assumptions of Savage's (1954) Sure Thing Principle and Anscomb and Aumann's (1963) Independence Axiom must be violated. Decades of research followed, analyzing and discussing how to adapt the axioms to best incorporate Ellsberg's (1961) insights about ambiguity. Important milestones were made by Schmeidler (1982, 1989), Gilboa (1987), and Chateauneuf et al. (2007) who in many ways complete the model. Eichberger and Kelsey (1999) provide an operational simplification in the form of Ellsberg capacities. Wakker (2010) provides a comprehensive overview of the relevant literature in the overall framework of decision-making under uncertainty.

For the purpose of this paper, it suffices to consider the most simplified utility function for ambiguity which still retains its characteristic features: it combines expected utility (weighted with the level of confidence γ) for "knowing" with a combination of the utility derived from "hoping for the best" (weighted by ω), "fearing the worst" (weighted by $1-\omega$), and for "not knowing" (weighted by $1-\gamma$). Thus, its structure is similar in spirit but extends the utility function proposed by Ellsberg (1961). It is in line with the discussion in Mukerji (1997) and with the utility function used in Spanjers (2008). This utility function simplifies the general framework of Chateauneuf et al. (2007) by what they call "simple neo-additive capacities" in the context of Choquet Expected utility. The resulting utility function for ambiguity looks as follows:

$$U(x, \pi, \gamma, \omega) = \gamma\, EU(x, \pi) + (1-\gamma)\, [\omega\, max_{s \text{ in } S}\, u(x_s) + (1-\omega)\, min_{s \text{ in } S}\, u(x_s)]$$

As before, x denotes, for $s = 1,\ldots, S$, the state contingent incomes $x = (x_1,\ldots, x_S)$. The decision maker's (subjective) probability estimate for the states is denoted by $\pi = (\pi_1,\ldots,\pi_S)$ and γ, with $0 \leq \gamma \leq 1$, denotes the decision maker's "level of confidence" in their (subjective) probability estimate π, with $1 - \gamma$ their lack of confidence. Furthermore, ω with $0 \leq \omega \leq 1$ denotes the decision maker's level of optimism, and $1 - \omega$ the decision maker's level of pessimism. Thus $\omega = 1$ denotes full optimism and $\omega = 0$ represents full pessimism. $EU(x, \pi)$ denotes the expected utility function for a vNM utility index u as before. The expression $max_{s \text{ in } S}\, u(x_s)$ denotes the highest utility for any of the state contingent incomes x_s with $s = 1,\ldots, S$ and represents hoping for the best. Similarly, $min_{s \text{ in } S}\, u(x_s)$ denotes the lowest utility for the state contingent incomes x_s with $s = 1,\ldots, S$ and represents fearing the worst.

From the utility function it immediately follows that the weight for each state $s = 1,\ldots, S$ equals $\gamma\, \pi_s$ except for the state with income x_{best} in which the highest utility is obtained and the state x_{worst} where the lowest utility is obtained.[4] The weight for the state s in which x_{best} is obtained equals $\gamma\, \pi_s + (1-\gamma)\, \omega$, in which the second term reflects the effect of the decision maker's "hope." Similarly, the weight for the state in which x_{worst} is obtained is $\gamma\, \pi_s + (1-\gamma)\, (1-\omega)$, in which the second term reflects the impact of the decision maker's "fear"

It should be noted that the states in which a decision-maker who faces ambiguity "hopes for" or "fears for" depend on the specific state contingent income $x = (x_1,\ldots, x_S)$ under consideration. As an example, for the state contingent incomes $x = (x_1, x_2, x_3) = (4, 5, 6)$, the decision maker hopes for state 3, with $x_3 = 6$, and fears for state 1 with $x_1 = 4$. For the state contingent incomes $y = (y_1, y_2, y_3) = (6, 5, 4)$, on the other hand, the same decision maker hopes for state 1, with $y_1 = 6$, and fears for state 3 with $y_3 = 4$. Thus, hope and fear shift weight (the counterpart of probability mass) to the states with the "best" and "worst" outcomes.

In general, the weights the decision-maker applies to the states $s = 1,\ldots, S$ to evaluate the state contingent incomes $x = (x_1, \ldots, x_S)$, may differ from the weights the same decision-maker applies to evaluate different state contingent incomes $y = (y_1, \ldots, y_S)$. If the weights are interpreted as subjective probabilities,

[4] Assuming that, for $x = (x_1,\ldots, x_S)$, the state with the best outcome and the state with the worst outcome are unique states.

we would be facing a "superstitious" decision maker, who assumes that the ranking of the outcomes over the different states affects the probabilities with which the states occur.

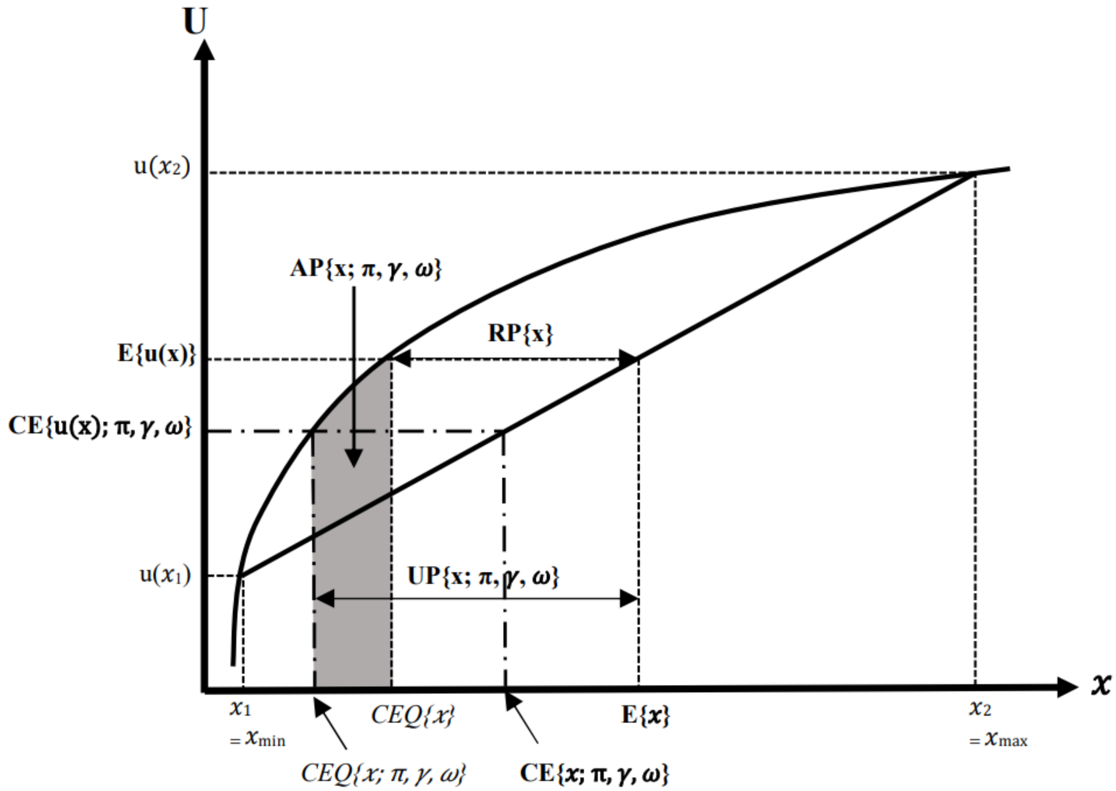
One may wonder how the intuition of the certainty equivalent *CEQ*, the risk premium *RP*, and the risk discount factor *DF* can be applied in the presence of ambiguity. For the certainty equivalent *CEQ*, this is straightforward because it is based on the same income *CEQ* being obtained for each state $s = 1,\ldots, S$ and, therefore, both the best case income and the worst case income equal *CEQ* too. This results in the certainty equivalent *CEQ* of (x, π) for the level of confidence γ and level of optimism ω, being obtained from $U(CEQ; 1, \gamma, \omega) = U(x, \pi, \gamma, \omega)$. Or, to put it differently:

$$u(CEQ) = \gamma\, EU(x, \pi) + (1-\gamma)\, [\omega\, max_{s\ in\ S}\, u(x_s) + (1-\omega)\, min_{s\ in\ S}\, u(x_s)].$$

To define the counterpart of the risk premium *RP* and the risk discount factor *DF*, we need to combine the certainty equivalent *CEQ* with the counterpart of the expected value of the ambiguous income. In the special case of neo-additive capacities that we consider here ("simple neo-additive capacities" in the terminology of Chateauneuf et al., 2007), it is natural to use the expected value of the prospect (x, π) for the counterpart of these benchmarks, for this purpose effectively ignoring the presence of ambiguity. Based on the certainty equivalent of the ambiguous prospect, the uncertainty premium of the prospect is defined as the difference between the expected income *EI* of the prospect and its certainty equivalent *CEQ*, that is $UP = EI - CEQ$. Similar to the case of the risk discount factor, the uncertainty discount factor for an ambiguous prospect now is obtained as $DF = CEQ / EI$.

One straightforward way to define the ambiguity premium *AP* of the ambiguous prospect would be to subtract the risk premium from the uncertainty premium: $AP = UP - RP$. For the situation with only two states, the uncertainty premium, the risk premium, and the ambiguity premium can be illustrated in the well-known diagram for the certainty equivalent and the risk premium, with $x_1 = x_{worst}$ and $x_2 = x_{best}$. This approach is followed in Diagram 1.

Diagram 1: Uncertainty Premium under Ambiguity



Source: Authors' illustration.

A different and more general way to illustrate the uncertainty premium and the ambiguity premium is to focus on the utility component of "not knowing" for (x, π) for the level of confidence γ and level of optimism ω:

$$\omega \; max_{\, s \; in \; S} \; u(x_s) + (1–\omega) \; min_{\, s \; in \; S} \; u(x_s).$$

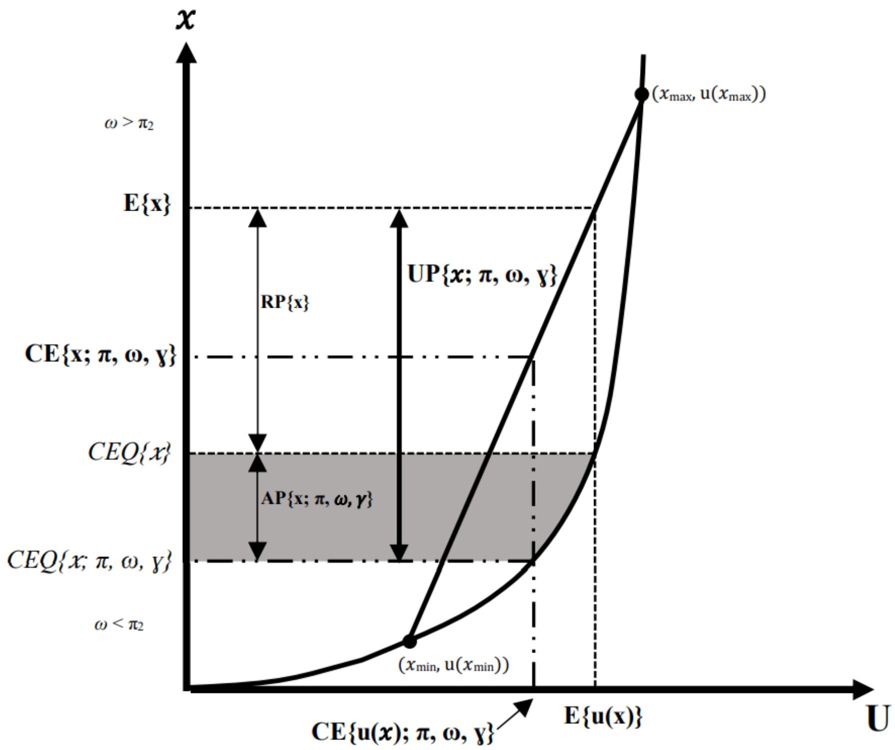
The only income levels to occur in this expression are x_{worst} and x_{best}. The utility component of "not knowing" with weight (1–γ) is combined with the expected utility component of "knowing" with weight γ as outlined before. The utility component of "not knowing" can be derived on the vertical axis of the standard type of diagram, replacing the probability π for x_1 by (1–ω) and the probability (1–π) for x_2 by ω.

The difference between the expected utility component for "knowing" and the utility of "not knowing" is positive when the former is larger than the latter. In Diagram 1, the utility component of "not knowing" can now be scaled by (1–γ) and subtracted from the expected utility value on the vertical axis. For this utility value under ambiguity, the associated certainty equivalent can be determined as usual. The certainty equivalent of the expected utility of the prospect can also be depicted as usual. The difference between the two now equals the ambiguity premium *AP* since

$$AP = UP – RP = (CEQ^{uncertainy} – EI) – (CEQ^{risk} – EI) = CEQ^{uncertainy} – CEQ^{risk}.$$

This is illustrated in Diagram 2, where the Choquet expected income **CE{*x*; *π*, *γ*, *ω*}** is less than **E{*x*}** if $\omega < \pi_2$ and **CE{*x*; *π*, *γ*, *ω*}** is larger than **E{*x*}** if $\omega > \pi_2$. Since **RP{*x*}** is independent both of the level of optimism ω and of the level of confidence γ, the change of these parameters on the ambiguity premium **AP{*x*; *π*, *γ*, *ω*}** follows directly from their effect on the certainty equivalent $CEQ\{x;\pi,\gamma,\omega\}$.

Diagram 2: The inverse of the Uncertainty Premium under Ambiguity



Source: Authors' illustration.

2.3 Prospect Theory

The Ellsberg Paradox is one of many behavioral biases that describe how real-life decisions may systematically deviate from the predictions of expected utility theory. Prospect theory provides a generalization of expected utility which incorporates the most common of these biases (Kahneman and Tversky 1979; Wakker 2010). The way it values obtained income reflects a dislike of underperformance, reflecting an emotionally driven misperception. It also incorporates misperceptions of the probabilities.

The structure of the initial standard model of prospect theory, however, creates the counter-intuitive possibility that a decision-maker may prefer a prospect providing a risk-free income, to a risky prospect that always increases this income. This property is known as the "violation of in-betweenness" and is considered a serious flaw in the theory. Cumulative prospect theory (CPT) introduced by Tversky and Kahneman (1992) convincingly mends the problem of the "violation of in-betweenness" (see also Wakker, 2010, and Kothiyal et al., 2014).

Our discussion of prospect theory starts with its two key building blocks: the value function and the probability weighting function. The value function v is the counterpart of the vNM utility index u. It provides a utility value to the gain or loss of income Δx compared to the decision maker's reference point R. The probability weighting function w transforms actual probabilities of the risky prospect $(\Delta x, p)$ into perceived probabilities $w(p)$.

Since prospect theory is based on gains and losses compared to a reference point, rather than actual levels of income, and on perceived probabilities rather than actual probabilities, prospect theory can be interpreted as a decision theory of distorted perceptions, i.e. of misperceptions. This leads to the following utility function of standard prospect theory:

$$V(\Delta x, p) := w(p_1)v(\Delta x_1) + \ldots + w(p_S)v(\Delta x_S)$$

where $(\Delta x, p)$ is a prospect with $\Delta x = (\Delta x_1, \ldots, \Delta x_S)$ and $p = (p_1, \ldots, p_S)$. For $s = 1, \ldots, S$ the difference between the income R of the reference point and the actual income x_s is denoted by $\Delta x_s = R - x_s$, whereas p_s denotes the probability of state s occurring.

After discussing the value function and the probability weighting function of perceived probabilities, we turn our attention to the utility function resulting from cumulative prospect theory. It turns out that the weights applied by cumulative prospect theory may resemble those obtained by Choquet Expected utility for decision-making under ambiguity (see Wakker, 2010). Reflecting a form of mental accounting based on the "compartmentalization" of gains and losses, (see also Thaler, 1980; 1999; Tversky and Kahneman, 1981; and Barberis and Huang, 2001), two Choquet integrals are applied separately: one to the valuation of gains and the other to the valuation of losses (see Wakker, 2010). We provide simple numerical examples to illustrate this and use it as a basis for interpreting prospect theory to combine an emotion-driven value function with knowledge-driven ambiguity.

2.3.1 The Value Function

In prospect theory, the value function is the counterpart of the utility index in expected utility. It denotes how outcomes are valued, if and when they occur. In the setting we consider, these outcomes are gains and losses of income compared to a target or reference level. Thus, a change in the reference value affects the perception of which levels of income are considered gains and losses as well as of the respective sizes of these perceived gains and losses.

The specific value functions considered by Tversky and Kahneman (1992) for prospects $(\Delta x, p)$ with $\Delta x = (\Delta x_1, \ldots, \Delta x_S)$ and $p = (p_1, \ldots, p_S)$ are structured as follows:

$$\begin{aligned} v(\Delta x_s) &= (\Delta x_s)^{\alpha} && \text{for } \Delta x_s \geq 0 \\ &= -\beta\,(\Delta x_s)^{\alpha} && \text{for } \Delta x_s < 0, \end{aligned}$$

where $\Delta x_s = R - x_s$ denotes the gain of income in state $s = 1, \ldots, S$ compared to the reference point R. For this type of value function, the degree of risk aversion is reflected in α, with $0 < \alpha < 1$,[5] and the degree of loss aversion is denoted by β, with $\beta > 0$.

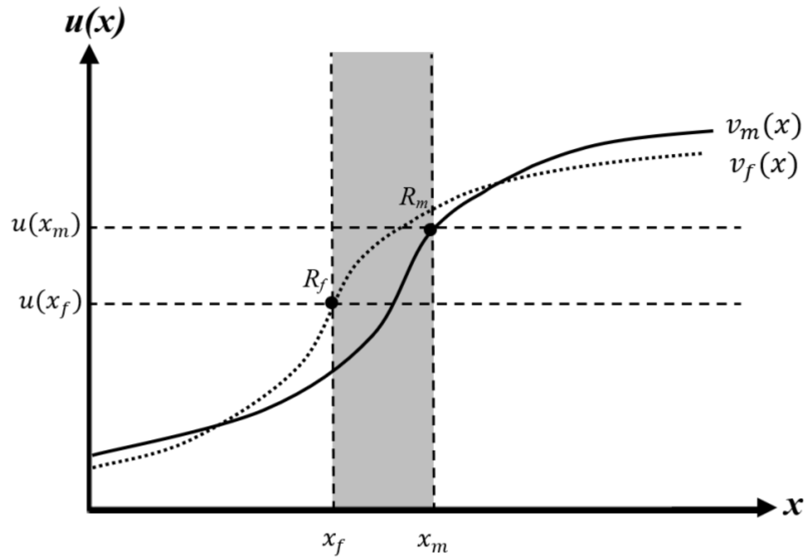
A key feature of this value function is that gains are treated differently from losses. The first part where $\Delta x_s \geq 0$ applies for gains is where the value function is concave. The second part where $\Delta x_s \leq 0$ applies for losses is where the function is convex. Thus, this type of value function represents a situation in which the decision maker is risk-averse concerning gains and risk-seeking when facing losses. A loss

[5] The degree of risk aversion reflected in α corresponds to a constant Arrow-Pratt coefficient of relative risk aversion of $(1\text{-}\alpha)$.

aversion parameter $\beta > 1$ causes the slope of the value for losses to be steeper than for gains of the same size.

Diagram 3 illustrates how a change of reference point affects the implied vNM utility index and how it affects the local risk attitude. In this example, we compare a prototypical ambitious male-gender-type decision maker with a high reference point R_m and a prototypical less ambitious female-gender-type decision maker with a lower reference R_f.

Diagram 3: Shift in reference points for opposite risk attitudes

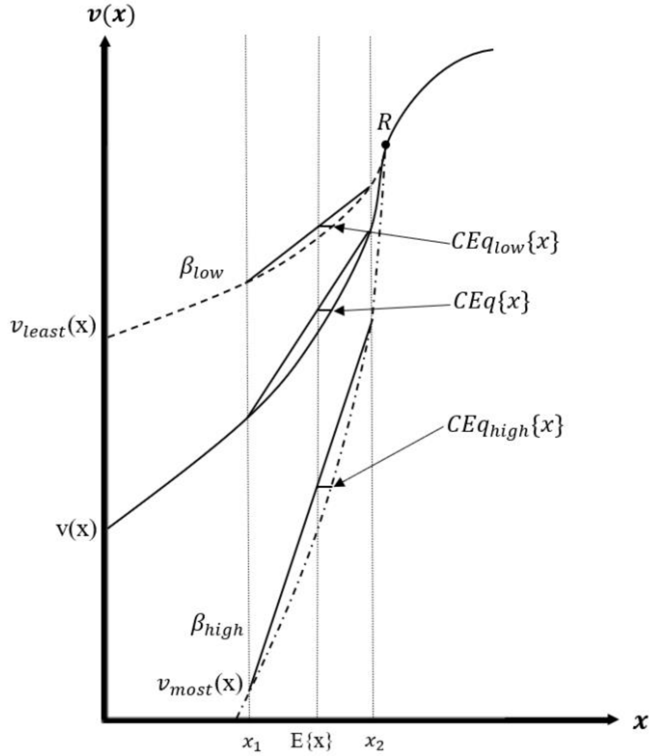


Source: Authors' illustration.

For a given value function v this leads to the vNM utility index $u_m(x) = R_m + v(x - R_m)$ for the more ambitious male-gender-type, and to the vNM utility index $u_f(x) = R_f + v(x - R_f)$ for the less ambitious female-gender-type. The former, $u_m(x)$, is represented by the solid curve, and the latter, $u_f(x)$, by the dashed curve. For the grey area in Diagram 3, between the reference points R_m and R_f, the male-gender-type implied vNM utility index $u_m(x)$ is locally risk-seeking, whereas the female-gender-type implied vNM utility index $u_f(x)$ is locally risk averse. This is easily verified by noting that for the grey shaded area, $u_f(x)$ is concave while $u_m(x)$ is convex. Since both vNM utility indices are based on the same value function v, the difference must be caused by the difference in reference point.

It should be noted that the equations for the utility indexes based on the value function v, are basically the same, the only difference being that $u_m(x)$ is shifted to the right and upward compared to $u_f(x)$ due to the change in the reference point for income from R_f to R_m.

Diagram 4: Change in the value function due to changes in loss aversion β.



Source: Authors' illustration.

Diagram 4 illustrates how an initial value function v is affected by changes in the decision maker's loss aversion. After a fall in loss aversion, from β to β_{low}, we end up with the value function v_{low}. After an increase in loss aversion, from β to β_{high}. For an undistorted perception of the probabilities, $w(p) = p$, and potential outcomes x_1 and x_2, Diagram 4 also depicts the associated certainty equivalents of the prospect (x_1, x_2; p_1, p_2) under standard prospect theory, with $x_1 < x_2 < R$, and with expected income **E{*x*}**. The certainty equivalent is the *x*-value of the right-hand end point of the line segment which denotes the risk premium.

Loss aversion only affects the part of the value function where the reference point is not being met and the levels of income are perceived as losses. In this part of the diagram, the decision maker is locally risk-seeking to try to "catch up" to their reference point. This is not affected by the intensity of the risk aversion as represented by α in the equation of Tversky and Kahneman's value function. Here, the loss aversion parameter β and the risk aversion parameter α do not affect each other, unlike what is suggested by Bouchouicha et al. (2019).

2.3.2 The Probability Weighting Function and Cumulative Prospect Theory

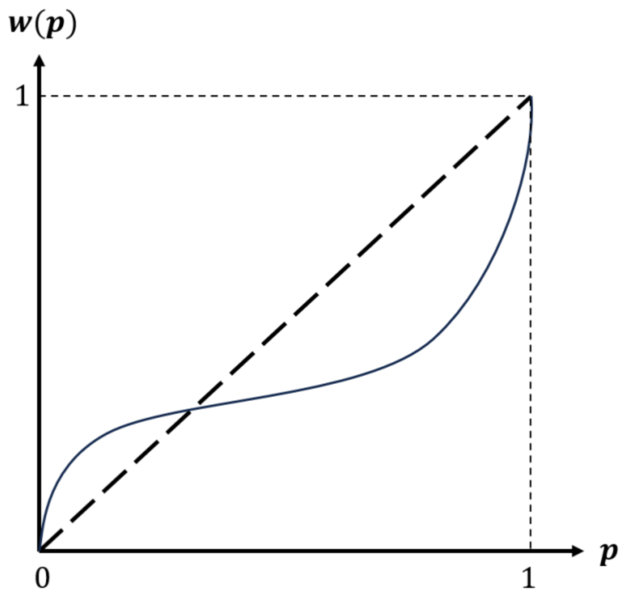
A familiar experience of everyday life is that decision-makers tend to have a biased perception of the probabilities of events. Typically, low-probability events are perceived to be more likely than they actually are. Similarly, high-probability events tend to be perceived as less likely than they actually are. In prospect theory, this systematic misperception of the probabilities is reflected in the probability weighting function w. Thus, the value $w(p)$ denotes the decision maker's perceived likelihood of an event occurring with actual probability p.

Tversky and Kahneman (1992) propose probability weighting functions of the following type:

$$w(p) = \begin{cases} 0 & \text{for } p = 0 \\ p^a / [p^a + (1-p)^a]^{1/a} & \text{for } 0 < p < 1 \\ 1 & \text{for } p = 1 \end{cases}$$

Their experimental data suggests the values for a to range from approximately 0.27 to 1, with an average of about 0.65. The typical shape of the probability weighting function is as in Diagram 5.

Diagram 5: The typical shape of the probability weighting function



Source: (Tversky & Kahneman, 1992).

One of the issues with the probability weighting function is that whereas the actual probabilities over all outcomes add up to one, this need no longer be the case for the perceived probabilities $w(p)$. Differences in the sum of the perceived probabilities may now lead to counterintuitive outcomes, like the "violation of in-betweenness" mentioned above.

Cumulative prospect theory resolves this issue by assuring that the same amount of weighting mass is assigned, irrespective of the prospect and the probability weighting function under consideration. The way this is achieved is by assigning separate weighting to losses, $\Delta x \leq 0$, and to gains, $\Delta x \geq 0$, in a way that ensures the sum of the weights to the losses is equal to one and that the sum of the weights to the gains is also equal to one. Thus, the total weight assigned under cumulative no longer equals one, as it would for probabilities, but rather equals two. Because the sum of the weights to losses equals one and the sum of the weights to gains equals one, the "violation of in-betweenness" – where the prospect's utility value is less than that of the highest loss or exceeds that of the highest gain – can no longer occur.

So how are these weights for losses and gains calculated? They are calculated by applying Choquet integrals to two adequately constructed non-additive probabilities (called "capacities"). These capacities are obtained by applying the probability weighting function to the "losses" part and the "gains" part, respectively, of the cumulative probability distribution of the prospect.

Loosely speaking, the gains are evaluated by applying the "pessimistic" Choquet integral to the capacity based on applying the probability weighting function *w*(*p*) to the cumulative probability distribution of the gains of the prospect. The losses, in contrast, are evaluated by applying an "optimistic" Choquet integral to the capacity based on now applying probability weighting function *w*(*p*) to the cumulative probability distribution of the losses of the prospect capacity (technically speaking, the "optimistic" Choquet integral applies the standard Choquet integral to what Chateauneuf et al. , 2007, call the "dual" or "conjugated" capacity of the capacity that is evaluated). For details, please see, for example, Tversky and Kahneman (1992) or Wakker (2010).

For simplicity, we restrict our attention to the probability weighting functions that lead to weights that have similar properties as the simple neo-additive capacities we focused on in Section 2.2 for ambiguity.

In part, these probability weighting functions effectively scale the actual probabilities down, with probability mass "leftover" being automatically assigned to the "no gains, no losses" outcome of zero. A further reallocation of weighting mass depends on the specifics of weighting function *w*(*p*). This further reallocation adds weighting mass to the largest gain which occurs with a probability larger than zero, the largest loss which occurs with a probability larger than zero, and the "no gains, no losses" outcome of zero. It should be noted that the weighting mass "leftover" is fundamentally different from the weights for not knowing associated with less than full confidence. As before in the case of the value function, the decision maker's reference point determines which actual outcomes are perceived as gains and which as losses. A change in reference point may, therefore, affect whether specific outcomes are evaluated through the 'optimistic' Choquet integral or through the 'pessimistic' Choquet integral, as used in Agliardi et al. (2022) to represent different attitudes towards a potential loss in biodiversity.

As an illustration, we consider a specific prospect and show the resulting distribution of cumulative prospect theory weights for the three different types of weighting functions specified below. In terms of ambiguity as in Section 2.2, for the standard "pessimistic" Choquet integral, applied for gains, the first type of weighting function leads to "full pessimism in gains" as is obtained under ambiguity for $\omega = 0$ (and, therefore, to "full optimism in losses" as for $\omega = 1$). Similarly, second type of weighting function leads to "full optimism in gains" as is obtained under ambiguity for $\omega = 1$ (and, therefore, to "full pessimism in losses" as for $\omega = 0$). The third weighting function leads to "a mixture of optimism and pessimism in gains" as is obtained under ambiguity for $0 < \omega < 1$ (and, therefore to "a mixture of optimism and pessimism in losses" as for $1 - \omega$).

The prospect (Δx; p) we consider for the three examples is as follows in Table 1:

Table 1: Numerical example for the three weighting functions

Δx	- 4.5	- 4.0	- 3.0	- 2.0	0	2	3	4	4.5
p	0.025	0.05	0.10	0.15	0.25	0.15	0.10	0.05	0.025

Weighting function with full pessimism in gains and full optimism in losses

The first type of weighting function, which leads to full pessimism in gains and full optimism in losses, is as follows:

$$w(p) = \begin{cases} 0 & \text{for} \quad p = 0 \\ \gamma \cdot p & \text{for} \quad 0 < p < 1 \\ 1 & \text{for} \quad p = 1 \end{cases}$$

where $0 \leq \gamma \leq 1$ is the scaling down factor, similar to the level of confidence in the context of ambiguity in Section 2.2.

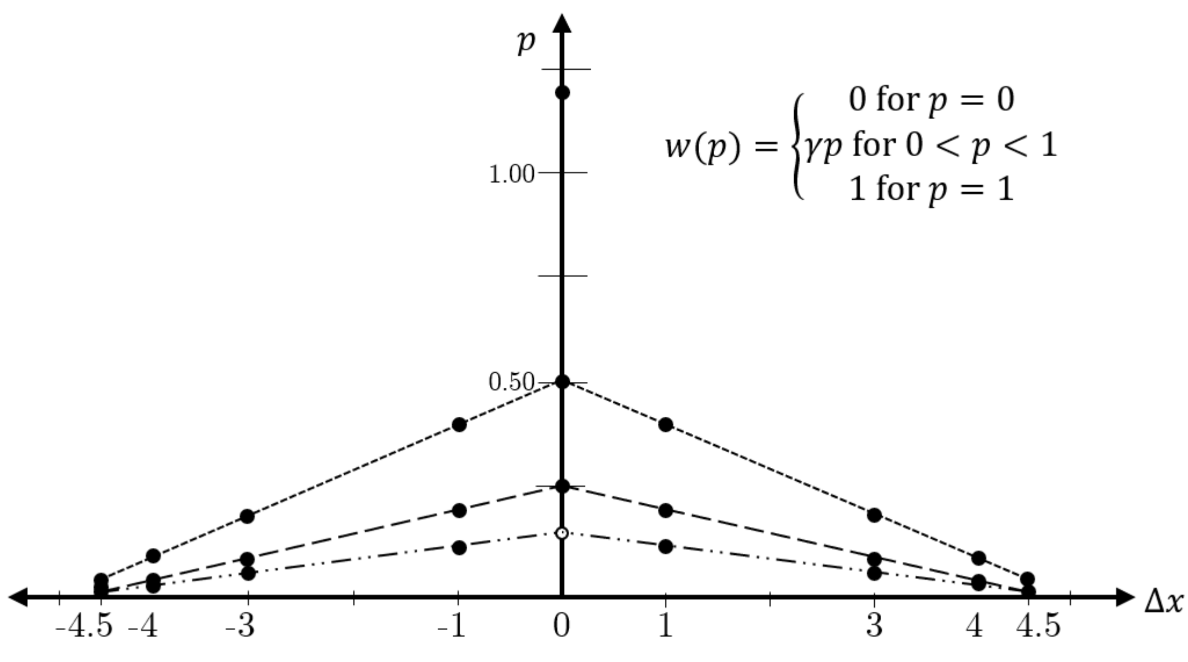
For the specific case of $\gamma = 0.8$, the cumulative prospect theory weights are as shown in Table 2.

Table 2: Cumulative prospect theory weights for the first type of weighting function

Δx	- 4.5	- 4.0	- 3.0	- 1.0	0	1	3	4	4.5
p	0.025	0.05	0.10	0.20	0.25	0.20	0.10	0.05	0.025
2*p*	0.05	0.10	0.20	0.40	0.50	0.40	0.20	0.10	0.05
CPT weight	0.02	0.04	0.08	0.16	1.40	0.16	0.08	0.04	0.02

To visualize the shifts in the weights obtained for this type of weighting function, the numbers for 2*p* and *CPT weight* are depicted in Diagram 6. There the small-dashed line with the highest weights is 2*p*; the large-dashed line with the second-highest weights is *p*, and the dashed-dotted-line with the lowest weights is *CPT weight*. Diagram 6 displays pessimism in gains by having the "leftover weight" of gains added to the weight of the lowest gain of zero. It also shows optimism in losses by having the "leftover weight" of losses added to the weight of the lowest loss of zero.

Diagram 6: The weights of the prospect with full pessimism in gains



Source: Authors' illustration.

Weighting function with full optimism in gains and full pessimism in losses

This second type of weighting function, with "full optimism in gains" and "full pessimism in losses" is as follows:

$$
w(p) = \begin{cases} 0 & \text{for} \quad p = 0 \\ (1-\gamma) + \gamma \cdot p & \text{for} \quad 0 < p < 1 \\ 1 & \text{for} \quad p = 1 \end{cases}
$$

where $0 \leq \gamma \leq 1$ is the scaling down factor, similar to the level of confidence in the context of ambiguity in Section 2.2.

For $\gamma = 0$, this weighting function puts a weight of 1 on the highest gain (in our example $\Delta x = 4.5$) and a further weight of 1 on the highest loss (in our example $\Delta x = -4.5$).

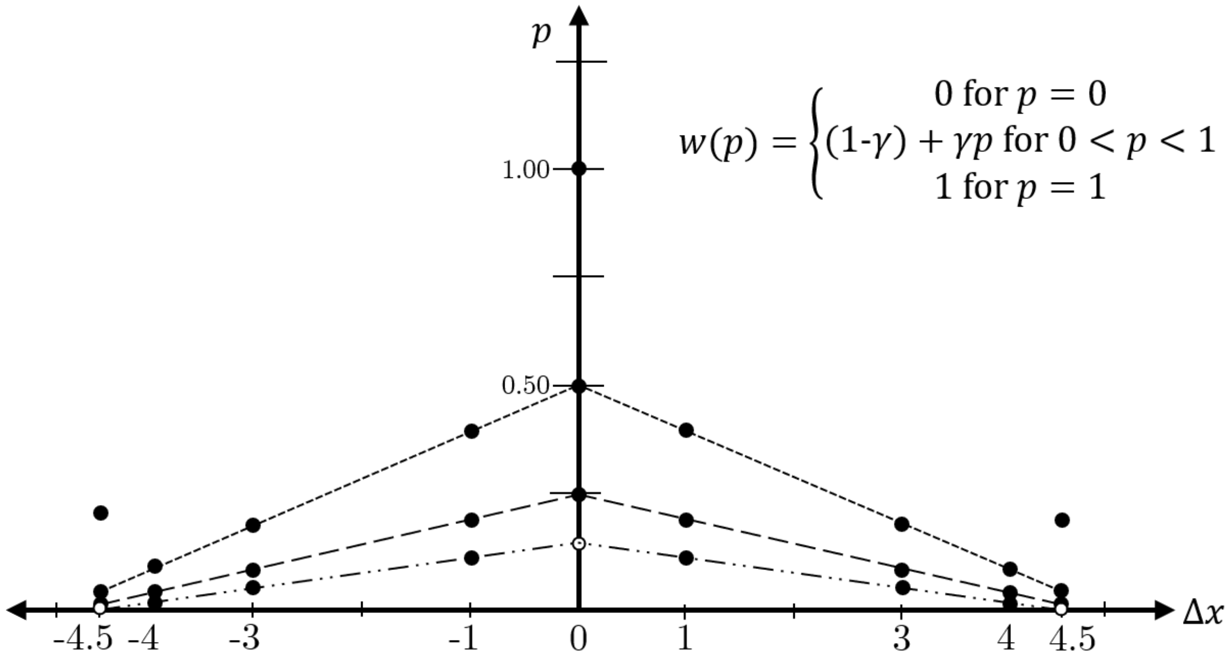
For the specific case of $\gamma = 0.8$, the cumulative prospect theory weights are as shown in Table 3:

Table 3: Cumulative prospect theory weights for the second type of weighting function

Δx	- 4.5	- 4.0	- 3.0	- 1.0	0	1	3	4	4.5
p	0.025	0.05	0.10	0.20	0.25	0.20	0.10	0.05	0.025
$2p$	0.05	0.10	0.20	0.40	0.50	0.40	0.20	0.10	0.05
CPT weight	0.22	0.04	0.08	0.16	1.00	0.16	0.08	0.04	0.22

To visualize the shifts in the weights obtained for this type of weighting function, the numbers for $2p$, for p, and for CPT weight are depicted in Diagram 7 as in Diagram 6. Here the weight of $(1-\gamma)$ is added to the highest gain (4.5) and to the largest loss (-4.5). The enhanced weight on "no gains, no losses" ($\Delta x = 0$) is due to the sum of the difference between the weight of the losses (excluding 0) and 1 as well as the difference of the sum of the weights of the gains (excluding 0) and 1; combined they make up the weight of "no gains, no losses" (i.e. 0).

Diagram 7: The weights of the prospects with full optimism in gains and full pessimism in losses



Source: Authors' illustration.

Weighting function with a mix of optimism and pessimism in gains and losses

This final type of weighting function has a mix of optimism and pessimism in gains and losses and is as follows:

$$w(p) = \begin{cases} 0 & \text{for } p = 0 \\ (1-\gamma)\cdot\omega + \gamma\cdot p & \text{for } 0 < p < 1 \\ 1 & \text{for } p = 1 \end{cases}$$

where $0 \leq \gamma \leq 1$ is the scaling down factor similar to the level of confidence in the context of ambiguity in Section 2.2 and reflects the extent of "not knowing." The parameter $0 \leq \omega \leq 1$ is similar to the level of optimism in the context of ambiguity in Section 2.2. For $\omega = 0$, the first type of weighting function with pessimism in gains and optimism in losses is obtained, for $\omega = 1$, the second type of weighting function with optimism in gains and pessimism in losses arises.

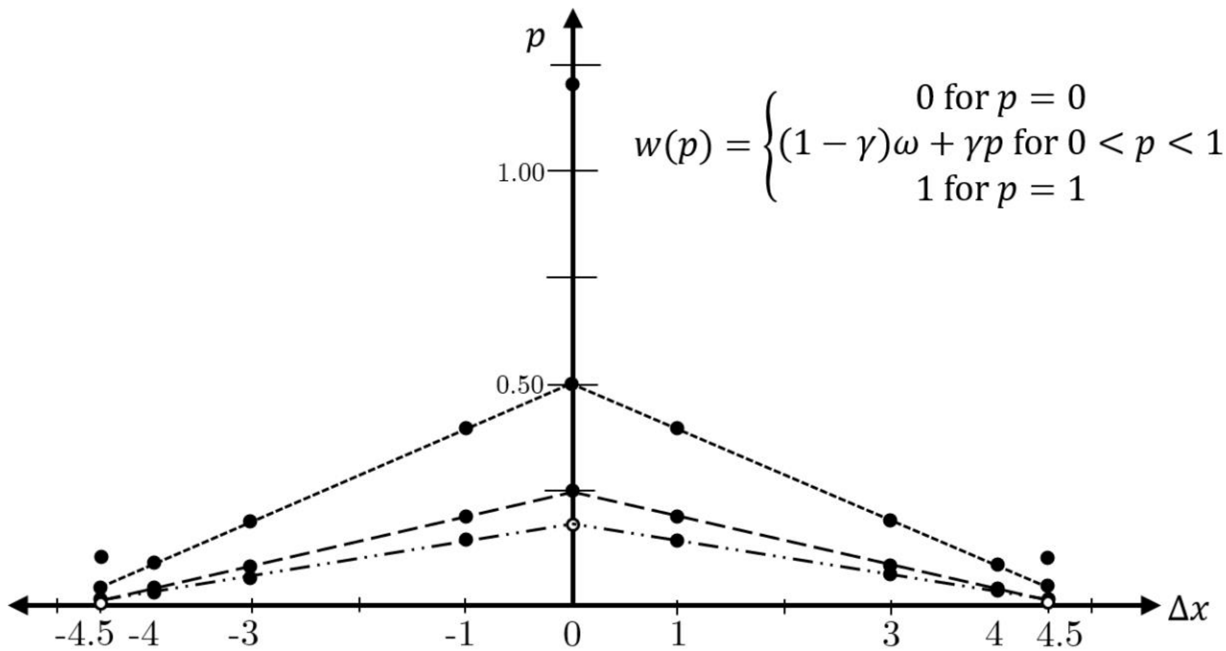
For the specific case of $\gamma = 0.8$ and $\omega = 0.5$, the cumulative prospect theory weights now become as shown in Table 4:

Table 4: Cumulative prospect theory weights for the third type of weighting function

Δx	- 4.5	- 4.0	- 3.0	- 1.0	0	1	3	4	4.5
p	0.025	0.05	0.10	0.20	0.25	0.20	0.10	0.05	0.025
$2p$	0.05	0.10	0.20	0.40	0.50	0.40	0.20	0.10	0.05
CPT weight	0.12	0.04	0.08	0.16	1.20	0.16	0.08	0.04	0.12

To visualize the shifts in the weights obtained for this type of weighting function, the numbers for 2*p*, *p*, and for CPT weight are depicted in Diagram 8, as in Diagrams 6 and 7 above.

Diagram 8: A mixture of pessimism and optimism in gains and losses



Source: Authors' illustration.

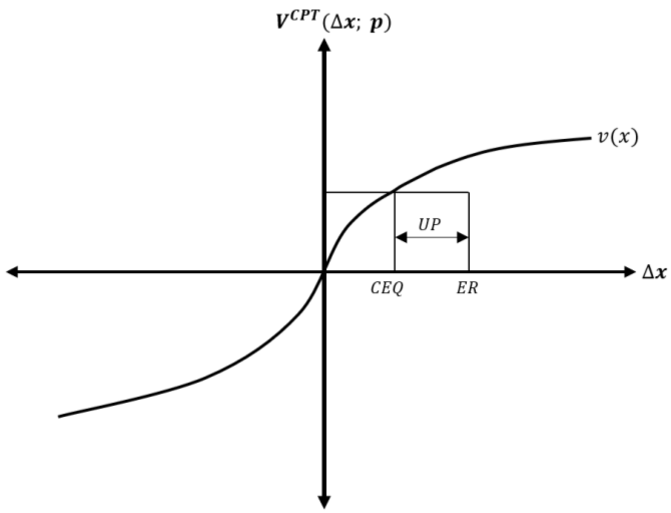
Here the weight of $(1\text{-}\gamma)\cdot\omega$ is added to the highest gain (4.5) and the largest loss (-4.5). The weight $(1\text{-}\gamma)\cdot(1\text{-}\omega)$ is added to the "no gains / no losses" (zero) outcome twice (once as a gain and once as a loss), where the "no gains/no losses" outcome is further enhanced by the difference between the sum of the weight of the losses (including zero) and one, as well as by the difference of the sum of the weights of the gains (including zero) and one. For the three types of weighting functions we consider here, these "leftover weights" are automatically assigned to zero. These leftover weights are different from the weights for not knowing associated with less than full confidence.

2.3.4. Certainty Equivalent, Premiums, and Discount Factor in Cumulative Prospect Theory

One may wonder how the intuition of the certainty equivalent, the risk premium, and the risk discount factor can be applied in the presence of cumulative prospect theory. For the certainty equivalent, this is straightforward because it is based on the prospect which pays a result ΔCEQ^{CPT} with probability one. The certainty equivalent of a prospect $(\Delta x, p)$ with $\Delta x = (\Delta x_1, \ldots, \Delta x_S)$ and $p = (p_1, \ldots, p_S)$ is the risk-free result $\Delta CEQ^{CPT}(\Delta x, p)$ that generates the same level of cumulative prospect theory utility as the prospect. That is, the cumulative prospect theory certainty equivalent ΔCEQ^{CPT} is such that $V^{CPT}(\Delta x, p) = V^{CPT}(\Delta CEQ^{CPT}; 1) = v(\Delta CEQ^{CPT})$. Although the total weight allocated by cumulative prospect theory is 2 rather than 1 as it would be for probabilities, this is not reflected in the equation for the certainty equivalent. Whenever ΔCEQ^{CPT} differs from zero it is either less than zero and obtains a "loss weight" of 1, or it is larger than zero and obtains a "gain weight" of 1. If ΔCEQ^{CPT}=0, it would receive a weight of 2, being the sum of 1+1 from the “loss weight” of 1 and the “gain weight” of 1. But this weight of two would be multiplied by $\Delta CEQ^{CPT} = 0$ and would, therefore, lead to the same utility as it would have for a weight of 1. For clarity, we denote the cumulative prospect theory certainty equivalent by $CEQ^{CPT} = R + \Delta CEQ^{CPT}(\Delta x, p)$.

To define the counterpart of the risk premium and the risk discount factor for cumulative prospect theory, we need to combine the cumulative prospect theory certainty equivalent, CEQ^{CPT} with the counterpart of the expected value of the result of the prospect (Δx; p), i.e. $ER^{CPT} = R + \mathbf{E\{\Delta x\}}$. Based on the cumulative prospect theory certainty equivalent of the prospect, the cumulative prospect theory uncertainty premium UP^{CPT} of the prospect is defined as the difference between the expected result of the prospect ER and its certainty equivalent CEQ^{CPT}, that is $UP^{CPT} = ER^{CPT} – CEQ^{CPT}$. Similar to the case of the risk discount factor, the uncertainty discount factor for cumulative prospect theory now is obtained as $DF^{CPT} = CEQ^{CPT} / ER^{CPT}$. This is illustrated in Diagram 9.

Diagram 9: The uncertainty discount factor for cumulative prospect theory



Source: Authors’ illustration.

3. On Modeling External Factors and Gender ‘Discount’ Factors

In Section 2 we considered internal bias discount factors, denoting the intensity of the devaluing of outcomes relating to various theories for decision making under uncertainty. We introduced and discussed the risk discount factor, the ambiguity discount factor, and the cumulative prospect theory discount factor respectively. One thing these discount factors have in common is that each relates to a taste or bias that is internal to the decision maker. They are internal factors.

In the current section, on the other hand, we consider factors that are external to the decision maker. External factors relate to biases in the environment that the decision maker operates in. These can be biases within markets or institutions or in the more general attitudes of society.

Whereas external factors tend to be related to gender, internal biases tend to be related to gender-types.

3.1 Gender 'Discount' Factors

The market's systematic bias against women's economic value as well as against the individual patience of female-gender-type entrepreneurs is well known. We propose to model this by extending the internal discount factors discussed in the previous subsections by an external discount factor which captures the distortions arising from the biased market.

This external market bias discount factor reflects the attitudes of investors that may prevent interested individuals from becoming entrepreneurs and create inequalities for different gender-types in their opportunities to succeed in the available career paths. At its heart, this is an issue of adverse selection, where investors try to select entrepreneurs with characteristics that they perceive to best "fit" their purpose. The entrepreneur's gender is treated as an indication that the person in question is more likely to have specific gender-type characteristics. Since, as mentioned previously, the male population and female population consist of both gender-type persons in different proportions, this may lead to unequal of persons of different gender with the same relevant characteristics. Evaluations based on stereotypes can be inaccurate and harmful (Cusack, 2013).

As an example, consider the internal intertemporal discount factor, which reflects the entrepreneur's patience with respect to future income. This intertemporal discount factor is a similar indicator of predicting success as grit, i.e. perseverance plus passion, in Duckworth et al. (2007). This internal intertemporal discount factor is known to the entrepreneur only, i.e. it is private information. Furthermore, this asymmetry of information arises before any contract is signed or agreement is entered. Thus, it is a case of "ex-ante" asymmetric information or, as it is better known, of adverse selection. This ex-ante asymmetry in information may create incentive issues, particularly in conjunction with "ad interim" asymmetric information problems, known as moral hazard problems. Such moral hazard problems are about providing incentives for people with known intertemporal discount factors to choose the desired behavior in situations of asymmetric information. Typically, in such situations, either relevant details of the situation are only known to the decision maker ("hidden knowledge"), or the decision itself is neither directly nor indirectly observable by others ("hidden actions").

Building upon this, we examine how external factors may affect an entrepreneur's internal intertemporal discount factor, making the entrepreneur behave less or more patiently. For example, harmful institutional practices or a lack of opportunities and resources, especially social and professional networks, may encourage women to behave more cautiously and more patiently, in line with the stereotypical female-gender-type, which reinforces gender norms and can contribute to gender inequality (Ozkazanc-Pan and Muntean, 2018). Typically, knowing the right people can greatly expedite entrepreneurial processes, especially in essential matters like accessing finance and starting projects. In addition, more years of education contribute to a stronger and more extensive network of connections, which could reduce the "costs" of women taking on "men's" roles, since the women would not have to engage in "catch-up networking" to a greater extent. Education is also seen as a credential in the selection process, causing unequal opportunities in education leading to further gender disparities.

3.2 Signaling

To unpack the issue of under-representation of women entrepreneurs, in this section we consider a selection process similar to that of Spence (1973). The setting is that of external investment in entrepreneurial firms either controlled by men or by women. Whereas the gender of the person in control of the firm is observable, the gender-type of this person is not. We now assume that female-gender-type entrepreneurs have discount factors reflecting patience and that male-gender-type entrepreneurs have discount factors indicating impatience. Furthermore, we assume that firms controlled by (impatient) male-gender-type entrepreneurs create more value for the investors than firms controlled by (patient) female-gender-type entrepreneurs. Since the gender-type is private information to the entrepreneur, firms may consider offering entrepreneurs the choice from a menu of different conditions for investment in the firm, in the hope that the different gender-types self-select through their choice of conditions.

To apply the basic principle of Spence's (1973) signaling model, we consider an offered contract C that provides women with a male-gender-type with a utility $U_{MW}(C)$ which exceeds the utility of "outside option" with the utility U_L which they receive if they reject the contract. At the same time, the contract provides the females with the female-gender-type with a utility $U_{FW}(C)$ that is lower than the "outside option". Hence, for women with a male-gender-type we have $U_{MW}(C) \geq U_L$ and for women with a female-gender-type we have $U_{FW}(C) < U_L$.

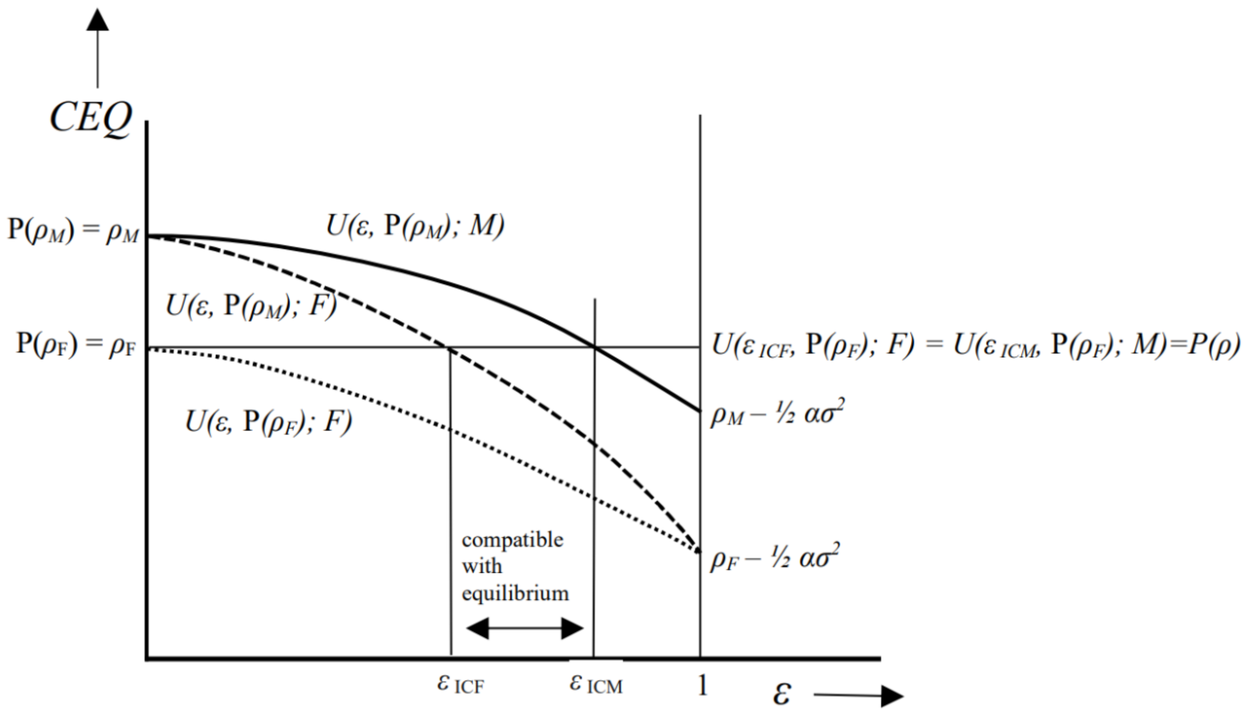
The application of the basic mechanism of Spence's (1973) signaling model as in Hirshleifer and Riley (1992), is illustrated in Diagram 10 below.

In Diagram 10, $\rho \in [0,1]$ is the expected return of an entrepreneur's project/business. Due to the gender-type discount factor for female-gender-type entrepreneurs, two levels of expected return are considered: low-return ρ_F for female-gender-type controlled firms and high-return ρ_M for male-gender-type controlled firms. The variance of each project is $\sigma^2 = 1$. The degree of absolute risk aversion is denoted by α. The fraction of the project retained by the entrepreneur is denoted by ε, and $(1 - \varepsilon)$ is the fraction sold to the investor.

The fraction of retained equity ε now serves as a costly 'signal' of the entrepreneur's gender-type. Retaining equity both exposes the entrepreneur to the actual return of the firm and inflicts a loss in the size of the risk premium. For moderately large fractions of retained equity ε, i.e. for $\varepsilon_{ICF} \leq \varepsilon \leq \varepsilon_{ICM}$ as in Diagram 10 below, the combination of the low expected revenue from the retained equity for female-gender-type controlled firms on the retained equity, $\varepsilon \cdot \rho_F$, and the risk premium due to the retained equity, $½ \cdot \alpha \cdot \varepsilon^2$, causes the female-gender-type entrepreneur's utility to fall below that of the outside option, U_L. For the same level of retained equity ε, however, the utility of the male-gender type entrepreneur remains above the utility of the outside option, due to the higher the high expected return of male-gender-type controlled firms, since $\rho_M > \rho_F$.

The horizontal line to the equity price $P(\rho_F)$ represents the incentive compatibility constraints. For fractions of retained equity ε for which the dashed $U(\varepsilon, P(\rho_M); F)$ curve is below the horizontal $P(\rho_F)$ line, the female-gender-type controlled firms will choose to sell all the equity in their firm at the price $P(\rho_F)$. For the fractions of retained equity ε where the dashed $U(\varepsilon, P(\rho_M); F)$ curve is above the horizontal $P(\rho_F)$ line, however, the risk-free payment $P(\rho_F)$ for selling a female-gender-type controlled firm will be less than the certainty equivalent of the female-gender-type controlled firm for retaining the fraction ε of the equity and selling the rest at the high price $P(\rho_M)$, that is $U(\varepsilon, P(\rho_M); F)$. Therefore, female-gender-type controlled firms will be better off mimicking the equity selling behavior of male-gender-type controlled firms, and the incentive compatibility constraint for the female-gender-type controlled firms will be violated.

Diagram 10: Adverse selection and the signaling of high-profit entrepreneurial firms



Source: Authors' illustration.

Similarly, the fractions of retained equity ε where the solid $U(\varepsilon, P(\rho_M); M)$ curve is below the horizontal $P(\rho_F)$ line, the certainty equivalent of male-gender-type controlled firms retaining a fraction ε of their equity and selling the rest at the high price $P(\rho_M)$, will be less than the risk-free income of selling the entire firm at the low price $P(\rho_F)$. Therefore, male-gender-type controlled firms will be better off mimicking the equity selling behavior of female-gender-type controlled firms, and the incentive compatibility constraint for the male-gender-type controlled firms will be violated.

The fractions of retained equity ε for which the incentive constraint of both the female-gender-type controlled firms and the male-gender-type controlled firms are satisfied are compatible with equilibrium. In Diagram 10, this is the case for fractions of retained equity ε in the range from ε_{ICF} to ε_{ICM}, which is labeled as 'compatible with equilibrium'.

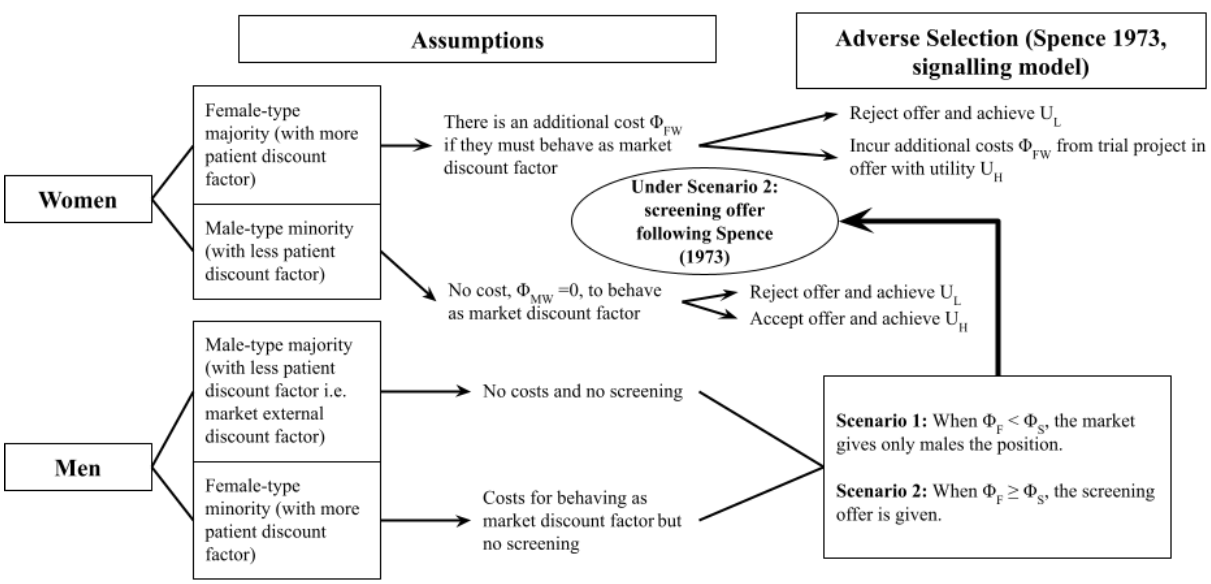
The version of the signaling model as in Diagram 10 applies when it is not possible or not allowed by law to take the actual gender of the entrepreneur into account in the investor's offered contract. If, however, it is possible to provide different contracts based on the observable gender of the entrepreneur, the situation becomes more complicated, as we discuss next, and is depicted in Diagram 11.

The offer C offered by the investor may, for example, have the requirement for women to implement a low-stake trial pilot project T, where women must behave according to the male-gender-type risk-loving and impatient characteristics. Taking part in this trial leads to additional utility cost of $\phi_{FW}(T)$

for women with female-gender-type and of $\phi_{MW}(T)$ for women with male-gender-type, with $\phi_{FW}(T) > \phi_{MW}(T)$. This difference could now result in $U_{FW}(C) - \phi_{FW}(T) < U_L \leq U_{MW}(C) - \phi_{MW}(T)$ as required in the signaling model above. For the investor, offering the trial pilot project involves a cost of $\phi_S(T)$. We assume the position is only given to women if an investor is willing to incur the screening cost $\phi_S(T)$ and some women are willing to incur the additional screening cost $\phi_{MW}(T)$ as required as part of the offered contract *C*.

This is illustrated in Diagram 11 below, where we assume there is the above-mentioned additional "cost" $\phi_{FW} > 0$ in the utility of female-type women entrepreneurs to behave as male-type entrepreneurs and that no such cost applies, $\phi_{MW} = 0$, in the utility of male-type women entrepreneurs. This aligns with the work by Chadwick and Dawson (2023) in which women may have a conflict in their identity that negatively impacts their mindset and feelings of authenticity. We also assume that all women have access to an alternative "outside option" with the utility U_L if they reject the contract offered by the investor. This outside option provides the same utility U_L irrespective of the woman's individual gender-type.

Diagram 11: Different gendered entrepreneurs, bias discount factors, and adverse selection.



Source: Authors' illustration.

We assume that there is a "cost of failure" ϕ_F to the investor when giving contracts to men only. The investor's "cost of screening" ϕ_S is total cost to the investor of screening all male-gender-type women, since the trial project is designed to ensure that it is only worthwhile for male-gender-type women to participate.

Under scenario 1, $\phi_F < \phi_S$, costs to the investor for not awarding contracts to women are less than the investor's total costs of screening. As a consequence, the market only offers men opportunities to become entrepreneurs. If the fraction of males with the female-gender-type is small, all males may be given the contract since investors would rather have a small number of failures than incur high costs of the screening process. Under such circumstances, female-gender-type men may become entrepreneurs, whilst even male-gender-type women will be denied this opportunity.

Under scenario 2, the opposite is true and $\phi_F \geq \phi_S$, costs to the investor of the failure to offer contracts to women are greater than or equal to the costs of screening all male-gender-type women. In this case, the market will offer women the screening option. Then it is up to women whether or not to choose the "outside option" with U_L. If they do not choose the outside option, they will have utility $U_{FW}(C) - \phi_{FW}(T)$ and $U_{MW}(C) - \phi_{MW}(T)$ for female-type women and for male-type women, respectively, due to their additional cost from the trial project, of $\phi_{FW}(T)$ and of $\phi_{MW}(T)$. Accepting such offer would not be attractive to female-gender-type women, but would it be attractive to male gender-type women. Thus, only male-gender-type women will participate in the low-stakes trial pilot project *T*.

As a consequence, under both scenarios, the female-gender-type entrepreneurs will all be men, not women. All men are being offered the opportunity to become entrepreneurs, due to the much higher fraction of male-gender-types in the male population than in the female population.

Proposition 1: ϕ_S, ϕ_F, ϕ_{FW} and ϕ_{MW} represent systematic bias discount factors or biases towards women. Regardless of the scenarios, there are fewer opportunities for women and only the minority will accept the offer.

This model shows that the opportunity for male-gender-type women to become entrepreneurs depends on the cost of failure ϕ_F that the investor is exposed to if contracts are only offered to men, for a given total screening costs ϕ_S. The higher the costs of failure ϕ_F in giving contracts to only men, the more likely the threshold is met for the investor to offer women the low stakes trial pilot project *T*, resulting in the male-gender-type women choosing to become entrepreneurs, and female-gender-type women choosing the outside option.

These qualitative results are consistent with intuition. They also reflect the fact that a large disparity in men versus women entrepreneurs is due to biased societal evaluation metrics depreciating women's achievements and outputs, especially when there is a misalignment or distortion of the gender-type discount factors, like the intertemporal discount factor considered here.

To further this argument, consider a second stage with moral hazard problem in which the principal *P*, the investor in the case of entrepreneurship, knows the agent, the entrepreneur, *A*'s risk characteristics and, based on this, provides them with incentives to the behavior that is in the best interest of investor. The "problem" lies in the investor's inability to obtain information about which state of nature the entrepreneur faces (in the case of "hidden knowledge"). The investor wants to maximize their profit from the contract and, therefore, would prefer not to incur costs in compensating for the risk premiums of female-gender-type entrepreneurs, where this can be avoided.

All in all, there are agency costs for overcoming asymmetric information in the two stages of adverse selection and moral hazard problems. These costs are integral to understanding the reason behind a gender-based overall discount factor that favors men and male-gender-type entrepreneurs over women and female-gender-type entrepreneurs. It puts an additional burden on both male-gender-type and female-gender-type women entrepreneurs and reduces their opportunities. In our policy implications, we suggest measures to correct this, for example by prohibiting discrimination based on gender in the contractual conditions. Such discrimination in the contractual conditions underlies the mechanism of the model in Diagram 11 and sets it apart from the model in Diagram 10.

3.3 Adverse Selection: Invisible Women in Family Firms

In this section, we consider the example of a family firm in a traditional society where the husband is the tie-breaker in decisions when the husband and the wife disagree. We focus on how gender roles and stereotypes play out in a family firm to influence decision-making. Furthermore, we assume the business environment is biased against women entrepreneurs as captured by an overall *gender discount factor* as discussed in Section 3.1.

As Smith (2023) states: "[Professor Goldin's] research illustrates how the process of closing the wage gap has been uneven throughout the course of history, changing in line with social norms and expectations about their career prospects and roles at home". Professor Goldin's argument that inequalities at home can sustain inequalities at the workplace is confirmed in our model of a family firm which faces adverse selection regarding its leadership, caused by unequal education opportunities of men and women.

Consider a husband and wife trying to start a family firm: one spouse has to play the "visible" role, i.e. taking the spotlight, while the other spouse plays the "invisible" role, i.e. doing important tasks in the background, but not representing the company's image as the entrepreneur. They must decide together based on each spouse's level of talent who will be the visible spouse and who will be the invisible spouse.

The wife knows her talents, but the husband does not know the talent of his wife unless it is above a threshold value for women's entrepreneurial talent to be detected, $\theta_{critical}$. This threshold value is determined by society and implemented through the educational system, reflecting unequal chances for women and men in education and development. The educational system ensures that when a woman's entrepreneurial talent is above the threshold this becomes verifiable, e.g. by issuing a degree certificate. The threshold value for women's entrepreneurial talent to be detected is what gives rise to the asymmetric information in the model. For women with entrepreneurial talent below the threshold value, the asymmetry in information is already present before any decisions on the running of the firm are made, so this is a situation of adverse selection.

We assume that the level of entrepreneurial talent among women is uniformly (or 'evenly') distributed from 0 (low) to 1 (high). The entrepreneurial talent of the wife is denoted by θ_W. We assume the same uniform level of talent for men and denote the entrepreneurial talent of the husband by θ_H. The focus of this model is on how the level of entrepreneurial talent of the wife, θ_W, affects the decision of who becomes the visible spouse.

This model is depicted graphically in Diagram 12 below, for a given level of entrepreneurial talent θ_H of the husband. In the diagram the line $q_M(\theta)$ denotes the expected success of the family firm when the husband runs the firm ("visible man") and the wife has entrepreneurial talent θ. Similarly, $q_F(\theta)$ denotes the success of the firm when the wife runs it ("visible woman").

When the entrepreneurial talent of the wife is above the threshold value, $\boldsymbol{\theta_W > \theta_{critical}}$, the entrepreneurial talent of the wife is certified and also known to the husband. So the husband and wife will agree whether it is in the best interest of the family firm to be run by the husband or by the wife.

Now consider the situation where the entrepreneurial talent of the wife is at or below the threshold value, i.e. for $\boldsymbol{\theta_W \leq \theta_{critical}}$. In this case, the wife's entrepreneurial talent is only known to the wife (it is 'private information' of the wife), and only she knows if it is better for the family firm to be run by her or by her husband.

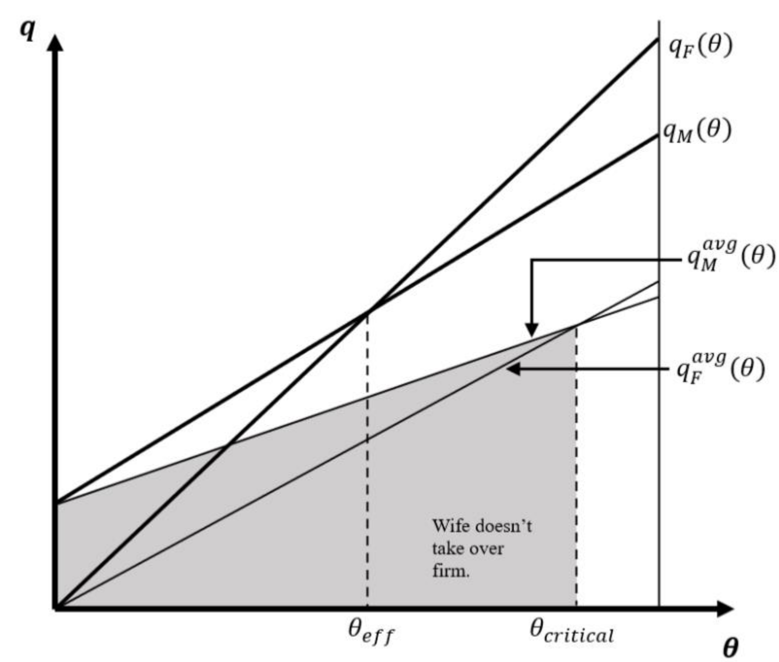
Consider the case where the husband is known to be of moderate talent. When the family firm now is now run by the wife it has greater potential for success when the wife's talent is high, but it also has greater potential for failure when the wife's talent is low. This is why the line $q_F(\theta)$ has a steeper slope than the line $q_M(\theta)$. The wife recognizes that it would be better for her husband to be the visible spouse if her own talent θ_M is below the value θ_{eff}.

The husband only knows the distribution of talent over all women and knows the threshold value for women's entrepreneurial talent to become publicly known, $\theta_{critical}$. But the husband does not know the talent of his wife. From the husband's perspective, the wife's talent now is uniformly (or 'evenly') distributed between 0 and the threshold value $\theta_{critjcal}$. From the husband's perspective, the expected success of the family firm when run by his wife is $q^{avg}{}_F(\theta_{critical})$, whereas it is $q^{avg}{}_M(\theta_{critical})$ when he runs it. These two lines are depicted in Diagram 12 too.

The wife will run the family if the wife and husband agree that this is in the best interest of the firm. When the entrepreneurial talent of the wife θ_W is below the threshold value $\theta_{critical}$, they will agree on this when both $q_F(\theta) > q_M(\theta)$, i.e. $\theta_W > \theta_{eff}$ (*wife agrees*) and $q^{avg}{}_F(\theta_{critical}) > q^{avg}{}_M(\theta_{critical})$ (*husband agrees*).

Let us now consider the situation depicted in Diagram 12. In the diagram, $\theta_{critical}$ is close to 1 and the intersection of $q^{avg}{}_F(\theta)$ and $q^{avg}{}_H(\theta)$ is slightly to the right of it, so $q^{avg}{}_F(\theta_{critical}) < q^{avg}{}_M(\theta_{critical})$. The *wife* will agree to run the firm if her talent is to the right of θ_{eff}. In the situation depicted in the diagram, this is the case whenever $\theta_W > \theta_{eff}$. The *husband* will agree to his wife running the family firm when *either* θ_W is to the right of $\theta_{critical}$ and he knows his wife is a highly talented entrepreneur, *or* $q^{avg}{}_F(\theta_{critical}) > q^{avg}{}_H(\theta_{critical})$, that is $\theta_{critical}$ is to the left of the intersection of these two curves. Since in the diagram, the point of intersection of the two curves is to the right of $\theta_{critical}$, the latter condition fails to hold and the husband will only agree to his wife running the family firm when $\theta_W > \theta_{critical}$

Diagram 12: Adverse selection and invisible women



Source: Authors' illustration.

Since here the husband and wife will only agree when θ_W is both larger than θ_{eff} and $\theta_{critical}$, and $\theta_{critical}$ is larger than θ_{eff}, the wife will run the family firm only when θ_W exceeds $\theta_{critical}$, that is when her entrepreneurial talent exceeds the threshold value for being detected by the educational system. Essentially, this is why the gender discount factor is embedded the model: the wife's talents are often unnoticed and underestimated, so she becomes the invisible wife.

4. Interpreting Behavioral Gender Differences

In the survey paper by Sent and van Staveren (2019), the behavioral economics literature on gender differences is organized using four key categories: risk appetite; overconfidence; trust; and altruism. In this section, we will discuss insights of the relevant empirical literature based on these four categories, and relate them to the decision-theoretic approaches of Section 2, as depicted in the diagram in Appendix 1, and to the interaction models of Section 3.

4.1 Risk Appetite

One of the fundamental questions we face is how the choices made by female-gender-type entrepreneurs in the presence of uncertainty differ from those of their male-gender-type counterparts. Typically, we evaluate the entrepreneurs' state-dependent outcomes using their beliefs and expectations in light of their appetite for uncertainty and, more specifically, for risk. Where the uncertainty takes the form of calculable risk, the entrepreneur's risk appetite is the tendency for decision-makers to prefer certain (risk-averse) or uncertain (risk-loving) outcomes, as outlined in Section 2.1. Evidence from empirical studies suggests that male-gender-types have a stronger appetite for risk than female-gender-types (Harris and Jenkins, 2006; Eckel and Grossman, 2008; Borghans et al., 2009; Hoang et al., 2019).

But as highlighted in behavioral decision models, calculable risk as discussed is not the only form uncertainty can take, and the behavior of decision-makers in the presence of uncertainty may differ strongly from what is suggested by the expected utility approach discussed in Section 2.1.

When considering situations of uncertainty characterized by ambiguity, partial knowledge, and limited confidence, the approach of Section 2.2 comes to bear. Here the gap in knowledge created by "not knowing" within bounds opens the door for the emotions of hope and fear to enter the decision-making. With regards to gender differences, it seems that in the face of ambiguity, female-gender-type decision makers weigh possible losses more heavily ("fear") than male-gender-type decision makers, who tend to strongly emphasize possible gains ("hope"). These differences are most apparent at the plausible extreme outcomes (Olsen and Cox, 2010).

In Section 2.2. the key aspects of this are captured by means of the level of confidence γ (with $0 \leq \gamma \leq 1$) and the degree of optimism ω (with $0 \leq \omega \leq 1$). The "lack of confidence", $1-\gamma$ dictates the extent of the decision maker's "not knowing", whereas the degree of optimism ω indicates their balance between hope and fear where they "do not know". It is shown how the interaction of the level of confidence and the degree of optimism impact the uncertainty premium. If γ is closer to zero, it allows more room for the probability estimates of the incomes to be biased by hope and fear through the degree of optimism, ω. This directly affects the value of the uncertainty premium as greater distortions in perception create a greater difference from $\boldsymbol{E}\{\boldsymbol{x}\}$, which results in a greater uncertainty premium value $\boldsymbol{UP}\{\boldsymbol{x};\boldsymbol{\pi},\boldsymbol{\gamma},\boldsymbol{\omega}\}$, as discussed in Section 2.2.

Clearly, it's important to differentiate between the risk premium, the ambiguity premium, and the uncertainty premium since they have different causes and respond differently to policy measures. The risk premium is caused by the risk attitude that arises from the curvature of the von Neumann-Morgenstern utility index u. The ambiguity premium, on the other hand, is caused by the attitude towards ambiguity, which arises from the degree of optimism ω. The uncertainty premium combines the risk premium and the ambiguity premium.

Even when considering situations of uncertainty that are fully described by (calculable) risk, the decision makers' behavior may be best described by models different from the expected utility, for example by prospect theory. As discussed in Section 2.3, prospect theory is a theory of misperceptions stemming from distortions created by emotions like an aversion to losing out compared to an individual reference point, called "loss aversion"). This loss aversion causes losses (outcomes below the reference point) to hurt twice as much as gains (outcomes above the reference point) of the same size give pleasure. Furthermore, the risk attitude of a decision maker following prospect theory depends on whether they are facing potential losses or potential gains. When facing potential losses, the decision maker is risk-seeking, trying to make up for the lost ground. When facing gains, however, the decision maker is risk-averse, trying to play it safe and secure the gain.

A prospect theory decision maker's appetite for uncertainty, therefore, depends on the intensity of the uncertainty about gains or losses, as well as on the intensity of their loss aversion. The certainty equivalent and the uncertainty premium for prospect theory are discussed in Section 2.3.4, as is the cumulative prospect theory discount factor which indicates the relative disadvantage the mis-perceptions of cumulative prospect theory cause, compared to the expected result of the prospect.

It may seem reasonable to expect female-gender-type entrepreneurs to have a lower loss aversion than male-gender-type entrepreneurs. Although this seems hold for some definitions of loss aversion, it is not confirmed in the empirical literature regarding cumulative prospect theory (see Bouchouicha et al., 2019). Indeed, Booij et al. (2010) find that what appears to be a stronger risk aversion for women is caused by women's probability weighting function "pessimistically" emphasizing bad outcomes, in combination with a stronger loss aversion (see also Booij and van de Kuilen, 2009). For various reasons, it also seems reasonable to assume that not only women, but also female-gender-type entrepreneurs have a lower reference point than men and male-gender-type entrepreneurs (see e.g. Kettlewell et al., 2023). Everything else being equal, these effects cause female-gender-type entrepreneurs to switch to risk-averse behavior at lower outcomes than male-gender-type entrepreneurs, creating the impression of a lower risk appetite.

As before, it is important to differentiate between the risk premium, the uncertainty premium under ambiguity, and the uncertainty premium for cumulative prospect theory. These different premiums have different causes and respond differently to policy measures. The risk premium is caused by the risk attitude that arises from the curvature of the von Neumann-Morgenstern utility index u. The uncertainty premium under ambiguity is affected by the attitude towards ambiguity, which arises from the degree of optimism ω. The uncertainty premium for cumulative prospect theory originates from misperceptions, amongst others arising from the aversion to losing out.

4.2 Overconfidence

Typically, overconfidence is considered to overestimate the outcomes one can achieve or of the likelihood achieving good outcomes. It can be useful in the initial start-up stage of entrepreneurship, but is also associated with firm failure in later stages of entrepreneurship (Invernizzi et al., 2017; Singh, 2020). Therefore, overconfidence can mean different things depending on the stage of entrepreneurship, even within each of the different decision theories discussed in Section 2,

In the context of expected utility as in Section 2.1, overconfidence may best be interpreted as a misperception of the random variable describing the calculable risk, by assuming better outcomes for the states of nature, by assuming excessive probabilities of states of nature with “good” outcomes and too low probabilities for states of nature with “bad” outcomes, or a combination of both. Formally, the random variable assumed by the decision maker is stochastically dominant of the first order over the actual random variable. As a consequence, the overconfident decision maker is deluded by thinking the expected utility of the choice is higher than it actually is. Female-gender-type entrepreneurs are generally considered to be less overconfident than male-gender-type entrepreneurs. At times this may result in female-gender-type entrepreneurs being more realistic in their evaluation of projects. However female-gender-type entrepreneurs who are underconfident may shy away from projects that would have been worth investing in.

The context of ambiguity as in Section 2.2 allows for two further fundamentally different interpretations of overconfidence. The description of ambiguity in this section is based on two key parameters, the level of confidence γ the decision maker has in their probability assessment π, and the degree decision maker’s degree of optimism ω, which reflects the balance between them hoping for the best and fearing the worst when the probability assessment π fails to apply. One interpretation of overconfidence now is that the level of confidence γ would be higher than the objective situation would justify. A second interpretation of overconfidence is that the decision-maker applies a higher degree of optimism ω than would objectively seem reasonable.

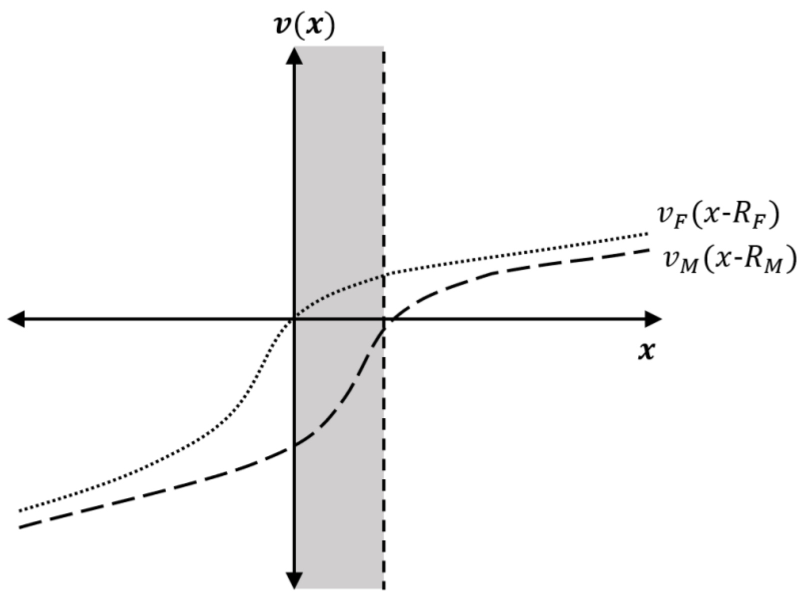
This constellation would lead the female-gender-type entrepreneurs to value projects with uncertain outcomes lower than their male-gender-type counterparts. As noted above in the context of expected utility, for the subjective probability estimate, π, of male-gender-type entrepreneurs uncertain projects tend to be valued higher than for the subjective probability estimate of female-gender-type entrepreneurs. Furthermore, female-gender-types are more pessimistic than male-gender-types and, for this reason, also put less emphasis on the best-case scenario — where their business is successful — too. This aligns with the narrative from the literature that women tend to be both more emotional than men and more pessimistic, and that entrepreneurs differentiate between confidence in the risky market and confidence in one’s own abilities (Barrett et al., 1998; Brebner, 2003; Chen et al., 2018; Felton et al., 2010; Puskar et al., 2014; Salamouris, 2013).

Research further demonstrates that men are generally more confident than women (Hügelschäfer and Achtziger, 2014; Sent and Staveren, 2019; Prasad et al., 2021). In the setting of ambiguity as in Section 2.2, this translates to female-gender-type entrepreneurs having a lower degree of optimism ω and a lower level of confidence γ, since a lower level of confidence leaves more room for emotions such as hope and fear to play a role. Clearly, the effect of the difference in the degree of optimism ω can be observed even when both gender-types have the same level of confidence $\gamma < 1$, or when both gender-types of entrepreneurs have the same degree of optimism ω but different values for the level of confidence γ.

So overconfidence as the combination of the distortion of the probability estimate π as for expected utility, and of male-gender-type entrepreneurs' higher level of confidence γ and a higher degree of optimism ω, will mostly combine for male-gender-type entrepreneurs to put a higher on value uncertain projects than female-gender-type entrepreneurs. As a consequence, female-gender-type entrepreneurs will decline to develop some projects which male-gender-type entrepreneurs are willing to develop, creating a disparity between the gender-types, and therefore the genders, of entrepreneurs.

In the context of cumulative prospect theory in Section 2.3, it is natural to interpret overconfidence as entrepreneurs overestimating their capabilities, being too ambitious, and, therefore, having an excessively high reference point. In other words, the entrepreneur is overranking his/her performance relative to others' performance (Bernoster et al., 2018). Diagram 13 below depicts the value functions of two entrepreneurs, one who is confidence-neutral and another who is overconfident. It seems reasonable to assume that also in this interpretation female-gender-type entrepreneurs are less confident and have a lower reference point than male-gender-type entrepreneurs (Kettlewell et al., 2023).

Diagram 13: Effect of overconfidence on different gender-type entrepreneurs



Source: Authors' illustration.

Let μ denote the extent to which the male-gender-type entrepreneurs are more (over)confidence than female-gender-type entrepreneurs, measured by the difference in their reference points R_M and R_F. The round-dashed line depicts a female-gender-type entrepreneur's value function v_F, whereas the long-dashed line depicts a male-gender-type entrepreneur's value function v_M.

As before with risk appetite in Section 4.1, when considering overconfidence, it is important to be aware of the different interpretations 'overconfidence' has in the context of expected utility in Section 2.1, in the context of ambiguity in Section 2.2, and in the context of cumulative prospect theory in Section 2.3. As in the case of risk appetite, these different interpretations reflect fundamentally different mechanisms and respond differently to policy measures.

4.3 Trust

Trust is understood to be one's belief in the trustworthiness of others, which can depend on a variety of signals (Gambetta, 2000; Jordan et al, 2016). Since trust is about beliefs, it can best be understood in the context of the decision models in Section 2, each of which may provide different insights, even though there is also an important intertemporal dimension to trust (e.g. social capital as in Spagnolo, 1999; Liu and Spanjers, 2009).

When considering the expected utility approach as in Section 2.1, one of the key questions is if and how the decision maker's risk attitude affects their trust in others in the context of the "trust game". The standard trust game, for example in Li et al. (2019, Figure 1), is a game in which the first player called the "truster" decides between choosing between distrusting or trusting the other player, called the "trustee". If the truster chooses to trust the trustee, the trustee chooses between "reciprocate", "middle", and "selfish". If the truster chooses to not trust the trustee, the interaction ends. In the case of distrust, the resulting payout is 10 for the truster and 10 for the trustee, respectively, denoted as (10, 10). When the trustee chooses to trust the trustee, the payouts are (15, 15) if the trustee reciprocates; (10, 18) if the trustee chooses "middle"; and (8, 22) if the trustee chooses to be selfish.

In the context of expected utility, the key papers of Eckel and Wilson (2004) and Houser et al. (2010) do not find an impact of the risk attitude on trust. Through this channel, there would be no reason to expect entrepreneurs of different gender-types to systematically behave differently with respect to trust.

This changes when the effects of ambiguity are taken into account. In the context of the trust game, the sophisticated analysis of Li et al. (2019) find an impact of the ambiguity attitude on trust: stronger ambiguity aversion reduces trust. They interpret the deviations from additive subject probability beliefs as an ambiguity attitude. In the context of the model of ambiguity in Section 2.2, this means that a higher degree of optimism ω tends to increase the decision-maker's trust, whereas a lower degree of optimism tends to make decision-makers less trustful. In their analysis, the counterpart of the level of confidence γ is not part of the analysis, as implied by Li et al. (2019, Footnote 5).

When applied to the comparison of different gender-type entrepreneurs, the result of Li et al. (2019) aligns with the established literature. As mentioned in Sections 4.1 and 4.2, female-gender-type entrepreneurs are less optimistic than male-gender-type entrepreneurs. The result of Li et al. (2019) now implies that female-gender-type entrepreneurs are less trusting than male-gender-type entrepreneurs. This is broadly in line with empirical studies, according to which female gender types are less trusting than male-gender-types (Sent and van Staveren, 2019).

Cumulative prospect theory as in Section 2.3 provides yet another perspective on trust. The issue of trust in the context of cumulative prospect theory is addressed Nguyen et al. (2016), who find in Result 2 that a strong loss aversion tends to reduce trust, as does a less distortive probability weighting function.

For the prospect in the examples of Section 2.3, the result of Nguyen et al. (2016) on loss aversion points in the same direction as the result of Li et al. (2019) on ambiguity aversion: lower loss aversion makes it more attractive for the truster to choose "trust" in the trust game, and the same holds when the truster has a higher degree of optimism. The counterpart of the less distortive probability weighting function reducing trust, that is of a higher level of confidence γ reducing trust, is analyzed by Li et al. (2019).

As in the case of ambiguity, the result of Nguyen et al. (2016) on loss aversion is broadly in line with the literature on gender and trust. Some results regarding cumulative prospect theory considers female-gender-type entrepreneurs to be more loss-averse than male-gender-type entrepreneurs (Booij et al., 2010; but also Bouchouicha et al. 2019) for which the result of Nguyen et al. (2016) implies that female-gender-type entrepreneurs are less trustful than male-gender-type entrepreneurs, as reported in Sent and van Staveren (2019).

In the model of asymmetric information in the family firm, in Section 3.3, trust can help achieve the efficient outcome, as the situation the husband and the wife find themselves in has similarities with the trust game. The husband can either decide based on his own limited information or leave the decision of whether he or his wife becomes the visible entrepreneur to his wife, who has full information. The former is the counterpart of the husband choosing "distrust" in the trust game, whereas the latter is the counterpart of choosing "trust". If the husband chooses to trust his wife, she has the possibility to make the decision in the best interest of the family firm, to "reciprocate", or to "selfishly" decide according to her own personal ambitions. When the husband "trusts" his wife and she "reciprocates", the outcome will be based on the wife's full information and will be in the best interest of the family firm.

4.4 Altruism

At its heart, altruism is about behaving selflessly, either entirely selflessly or selflessly within bounds. In the case of less than full selflessness the question arises what part of the available resources the decision maker should allocate altruistically, in what way, and to what purpose. In such situations, as in Andreoni and Vesterlund (2001), one may end up with some kind of 'selfish selflessness'. In the context of entrepreneurship and gender types, it makes sense to consider a decision maker's degree of altruism as the extent to which this decision maker takes the external effects of their actions into account, rather than the utility levels of others (see also Bester and Güth, 1998, and Cason et al. 2022).

When considering differences in levels of altruism between gender types, it is well established in the literature that women tend to behave more altruistically than men (Mellstrom and Johannesson, 2008). However, the level of altruism depends on access to resources. According to Andreoni and Vesterlund (2001), women on average behave more altruistically than men when the costs of behaving altruistically are high, while men are more altruistic when the costs of being altruistic are low. As an example, such "high costs of altruism" are like giving proportionally higher tips when the restaurant bill is large, which is consistent with the work by Parrett (2023).

Many studies suggest that being altruistic towards their management team, relatives, suppliers, and investors is a critical characteristic of successful leaders and entrepreneurs (e.g. Mallén et al., 2019; Alnajjar and Hashim, 2020). More generally, being perceived to be altruistic to people with no relation to the firm can be an important part of socially responsible corporate behavior.

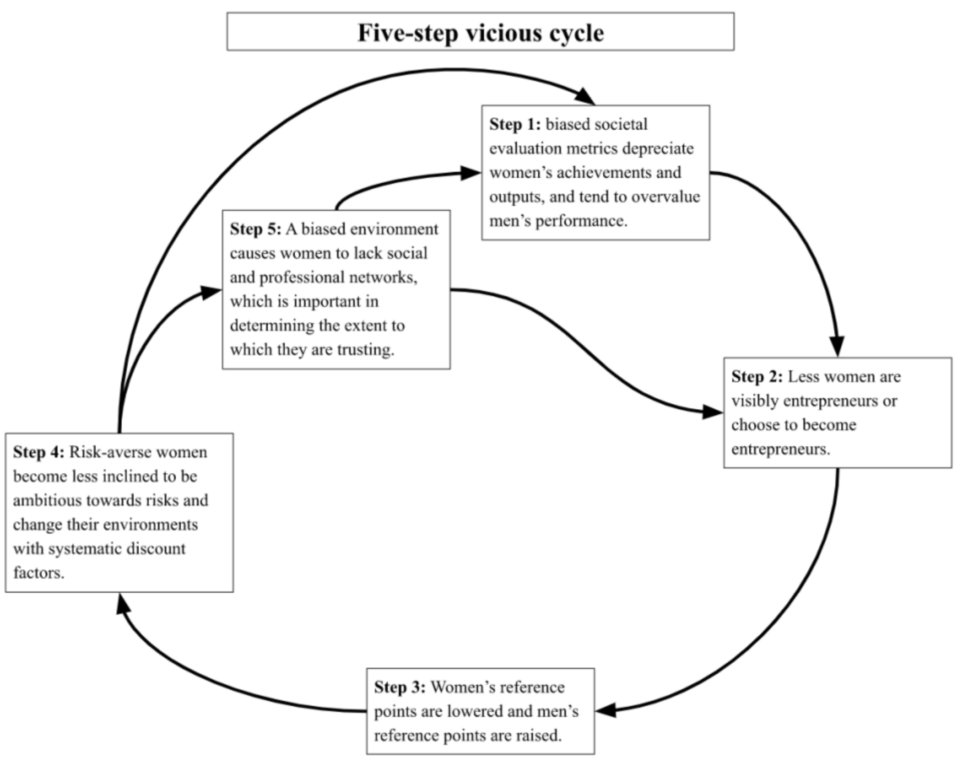
When it comes to receiving altruism, however, according to Urbig et al. (2012), people willing to exploit opportunities in investment and wage employment are more "entrepreneurially talented", suggesting that start-ups in their early stages face financing constraints and would be more likely to be net recipients of altruism.

In the setting of the signaling of entrepreneurial skills, as depicted in Diagram 11 in Section 3.2, one of the potential outcomes is that the only female-gender-type entrepreneurs are male. Investors may want to mend such situations which put female gender type women at a disadvantage compared to their male counterparts, as part of their corporate social responsibility. This could be achieved by requiring both men and women to successfully complete low-stake trial pilot projects to gain access to financial resources, even when it would be more cost-effective to require such projects only of women. The investors selflessly incurring the additional screening costs of this measure could, perhaps, be considered an act of altruism.

5. The Vicious Circle of Women Entrepreneurship

In this section, we provide a discursive framework connecting key gender barriers in entrepreneurship into a five-step "vicious circle of inequality" for women entrepreneurs. We summarize the insights gained through this vicious circle in seven findings. The five-step vicious circle is illustrated in Diagram 14 below.

Diagram 14: Barriers to women entrepreneurship: a five-step vicious circle



Source: Authors' illustration.

Step 1: Biased societal evaluation metrics depreciate women's achievements, and tend to exaggerate men's performance.

This step is summarized by the gender discount factor (see Section 3.1). Chamorro-Premuzic (2019) suggested that if leadership and business-ownership continue to be considered more masculine, female leaders will be evaluated negatively even when their performance is higher than that of their male counterparts. The gender discount factor reflects the environment the women entrepreneurs are in relative to a fair environment, which offers fair assessments, sufficient finance, and equal education, as well as enables access to networking and positive or negative communities that the decision-maker identifies with. The contribution of women entrepreneurs is underrepresented and is devalued in comparison to those of their male counterparts, especially when an evaluator is a man (Eagly et al., 1992).

Finding 1: *The disadvantages decision-makers face in their environment relative to a fair environment can be summarized in a bias discount factor. The fair environment would offer fair assessments, sufficient finance, and equal education, as well as enabling access to networking and positive or negative communities that the decision-maker identifies with.*

A higher level of education mitigates the biases in the evaluation of skills and abilities, increasing the chance that women generate innovations, especially technical-related ones (Fu et al., 2020). This observation regarding education levels is consistent with the European Commission's report (2016) that women are under-represented among the population of entrepreneurs. Women tend to operate smaller and less dynamic businesses than men and are more likely to operate in non-capital-intensive sectors including personal services. These sectors often have a lower potential for generating a high and sustainable income, but may be more successful in "socioemotional career satisfiers" that are not typical metrics that are factored into the definition of an entrepreneur's success (Chadwick and Dawson, 2023; Hechavarria et al., 2017).

Moreover, interesting observations arise on the intersections of gender, education, race, and immigrant status in technology entrepreneurship. This provokes discussion on a fundamental question in behavioral decision-making: people are hesitant to changes which clash with their identity, which both prolongs the existence of social discount factors and hinders the victims from escaping their consequences.

Step 2: Women entrepreneurs are less visible, and fewer women choose to become entrepreneurs.

Here mainly internal factors in decision-making are at play.

Finding 2: *The combination of lower confidence and a lower optimism regarding the uncertain decision to become an entrepreneur, makes female-gender-types less likely to pursue entrepreneurship, compared to male-gender-types.*

Essentially, the uncertainties and mis-perceptions about whether it is possible to prosper as an entrepreneur makes female-gender-types less likely to become entrepreneurs and, therefore, make it less likely for women to become entrepreneurs too.

Step 3: The relative lack of female role models lowers women's aspirations, whereas an abundance of male role models raises the aspirations of men.

Finding 3: *The aspirations can be changed by role models who act as reference points.*

When the majority of entrepreneurial role models are male, men's reference points and aspiration levels are raised compared to those of women. As a further consequence, especially in third-world countries, women may not even consider pursuing entrepreneurship as a plausible future option, because women rarely succeeded before. This is one of the reasons women in developing countries are the least likely to become entrepreneurs (Vossenberg, 2013). In sub-Saharan Africa, for example, women face a range of further institutional obstacles that stop them from participating in economic activities like entrepreneurship (Ekesinoye and Okolo, 2012).

Step 4: Lower aspirations reduce female entrepreneurs' appetite for uncertainty, increasing the gender discount factor.

Step 4 is best illustrated by prospect theory's reference point and value function, although the link between aspirations and appetite for uncertain projects also applies to other decision theories. For expected utility it may arise through the subjective probability distribution. For decision making under ambiguity, it may be caused both by the degree of optimism and by what are perceived to be the plausible best-case and worst-case outcomes.

Finding 4: *Aspirations affect risk attitudes, causing female entrepreneurs to avoid risks that male entrepreneurs embrace*.

The effects outlined in Steps 2 to 4 further decrease the gender discount factor, reflecting the fact that the external environment becomes even less supportive than it was before, taking the circle to Step 1 of the next iteration. Alternatively, it may proceed to Step 5.

Step 5: Women are less trustful than men, reducing the quality and effectiveness of their social and professional networks.

This step can be an alternative to Step 1 and lead back directly into Step 2. It is included in this "vicious circle" to highlight the role of communal behaviors like trust and altruism. Networking, for example, plays a crucial role in successful entrepreneurship and growth (Bogren et al., 2013; Manolova et al., 2017; Neumeyer et al., 2019). This step emphasizes that female entrepreneurs' lower levels of trust restrict their tendency to cooperate with more trustful male entrepreneurs in typical repeated Prisoner's Dilemma situations.

Finding 7: *Their stronger loss aversion and their higher pessimism, cause women to be less trustful than man. Being less trusting reduces the quality and effectiveness of their social and professional networks.*

6. Conclusion and policy implications

This paper addresses the question of why there are fewer female than male entrepreneurs in innovative sectors from the perspective of microeconomic theories. For decision-making under uncertainty we use appropriate accessible simplifications of the approaches of expected utility, ambiguity as in Choquet expected utility, and cumulative prospect theory. For the interactions between decision-makers, we use signaling and a tailor-made model of adverse selection. For our analysis, we combine gender-types differences with underlying uncertainty, which explores a new angle in gender and business studies. The proposed theoretical framework encompasses both internal and external factors that affect decision-making — such as risk appetite, overconfidence, trust and altruism — and relates them to decision theoretical concepts like risk attitudes, levels of confidence, optimism and pessimism, reference points, probability distortions, and a systemic discount factor reflecting the biased environment. We explore these different aspects of uncertainty to find ways to unify them, to connect them, and to differentiate between them. Appendix 2 outlines the theoretical framework and illustrates the interlinkages and connections of these models through specific aspects of expected utility and bias discount factors.

Our findings contribute to the literature in several ways. First, to the best of our knowledge, this is the first study that systematically analyzes the gender-type differences in entrepreneurial and innovation behavior, using a set of uncertainty theories from behavioral economics. Secondly, this paper sheds light on the roles of the entrepreneur's environment in shaping the gender-innovation relationship through education, legal protection acknowledging the economic value of entrepreneurs, access to finance and social services, and attitude to uncertainties. Thirdly, we use a systemic bias discount factor to capture the individual proportional disadvantage caused by systematic biases. We analyze the contingent role of such systematic biases in assessment in the gender-innovation relationship, as a useful first step in our attempt to understand the gender and gender-type difference in innovative firms managed by female and male entrepreneurs. Firms managed by women are devalued amongst other due to stereotypes towards women and female-gender-type characteristics. Lastly, this paper models key barriers to women's entrepreneurship that are associated with uncertainties and ambiguity, which are interconnected to become a "vicious circle of inequality" for women entrepreneurs in innovation.

In terms of policy recommendations, we start with the observation that even in countries with strong gender equality, such as the USA and the UK, we still find serious gender disparities in entrepreneurship in technology and high-innovation areas like artificial intelligence (Farah, 2023). It seems that gender equality is not always sufficient to achieve (approximate) gender parity. In the terminology of our paper, at times "gender equality" needs to be supplemented by "gender-type equality." Our results show the impact of differences in gender-types in creating gender disparities. This suggests that it may not be enough to only consider gender-related policies, but that to address gender disparities it may be necessary to also consider measures that overcome gender-*type* disparities as captured by the gender-type bias discount factors. From this perspective it may, for example, prove helpful to encourage two separate sustainable channels from investors to outlets: one focusing on male-gender-type entrepreneurs and another focusing on female-gender-type entrepreneurs. Thus, we conclude that there needs to be a holistic approach to addressing the entire "vicious circle" that discourages women from becoming entrepreneurs, requiring measures affecting the entire range from investors to consumers.

Additionally, we suggest testing the theoretical models proposed in this paper with empirical data to arrive at a more detailed and tailor-made, holistic policy approach in this context. The study on entrepreneurial policies conducted by the OECD (2021) is a detailed guide to typical policies. Our study provides a theoretical basis for many of its policies and beyond its framework. OECD (2021) highlighted that legislation needs to ensure an unbiased, objective environment through legal protection (*addressing negative impacts from external bias discount factors*); promoting profiles of women entrepreneurs through award programs (*addressing low reference points*); and building proactive network structures in entrepreneurship training and mentoring programs especially designed to address behavioral changes (*changing risk attitude, increasing trusting-ness, and addressing low confidence and pessimism*).

However, the traditional public policy measures mentioned above, aimed at addressing barriers to entrepreneurship, may not be effective at reaching potential female entrepreneurs unless they consider underlying factors in asymmetric information, risk, ambiguity, and perceptions of risk in entrepreneurial decision-making, which interact with each other but also with the environment in different contexts. Goldin (2023) critisized "[,,,] These responses haven't worked to erase the differences in the gender pay gap. And they will never provide a complete solution to gender inequality, because they treat only the symptoms. […]". A more nuanced and multi-dimensional approach is required.

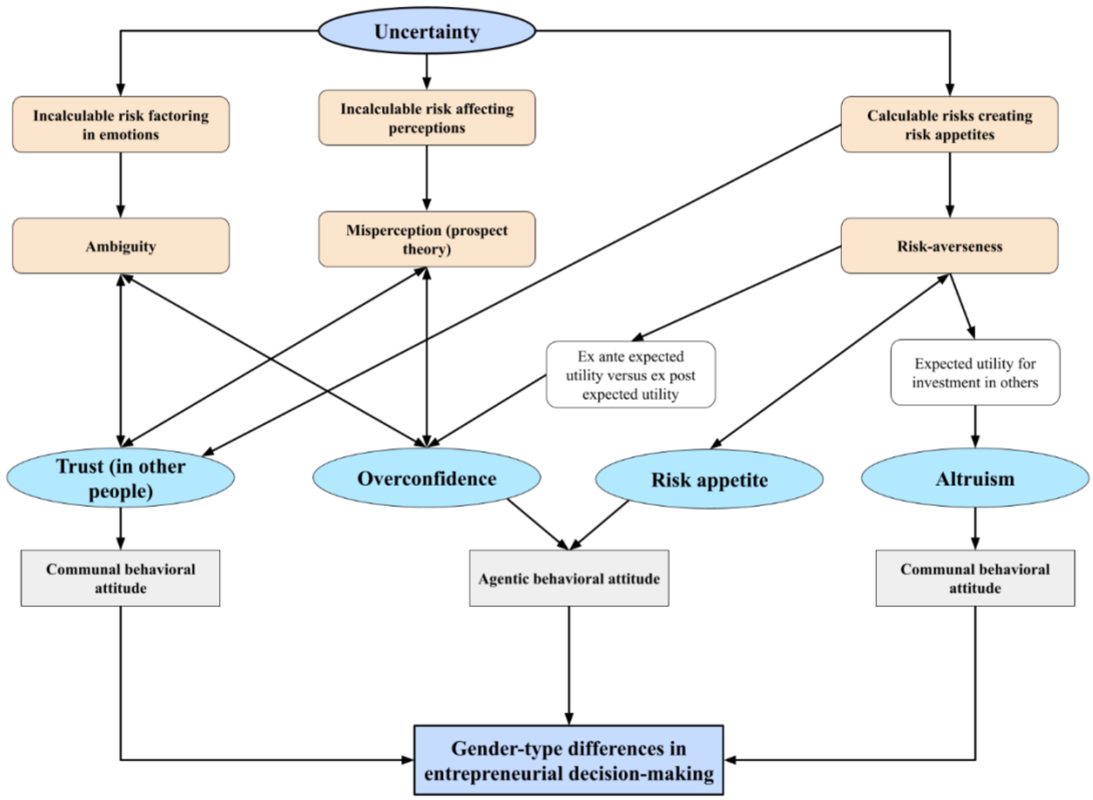
Such more effective approach should combine traditional gender policies with policies for long-term sustainability, encouraging firms to be more socially and environmentally conscious, promoting compassion and trust, and raising reference points for targeted marginalized groups. It would create a new business ecosystem that is more balanced and feminine-friendly, addressing gender biases that may not necessarily be associated with the development of a country or whether there is basic infrastructure to support entrepreneurs. For example, labels to indicate that a business is socially conscious and empowering women could be encouraged to nurture female and female-type entrepreneurship via public support and addressing issues with uncertainty.

Above all, it is necessary to have a multi-dimensional holistic policy framework that supports the establishment of an ecosystem for female and female-type entrepreneurs in which their style of entrepreneurship is uplifted. This simultaneously accelerates towards greater equality and sustainability, as suggested by the way female-type ("feminine") companies are more socially and environmentally conscious (Chadwick and Dawson, 2023; Eddleston and Powell, 2008; Hechavarria et al., 2017). We think that in the context of entrepreneurship this goes a long way to addressing the second question in the quote from Professor Goldin's in the introduction of this paper: "[…] why do women and not men step back from these higher paid opportunities? […] and how can we make these 'greedy jobs' less demanding, without making them less productive? That's the next frontier we have to explore." (Smith, 2023).

To summarize, there is a need for further empirical studies on the contribution of female-type companies led by women-entrepreneurs. Further theoretical analysis on the implication of a policy framework that incentivizes gender-type entrepreneurship through a deeper understanding of uncertainty perspectives is suggested. More research and policy deliberation in these areas would help fill knowledge gaps and would contribute to addressing fundamental problems that might bring us closer to gender equality in a sustainable manner in different contexts.

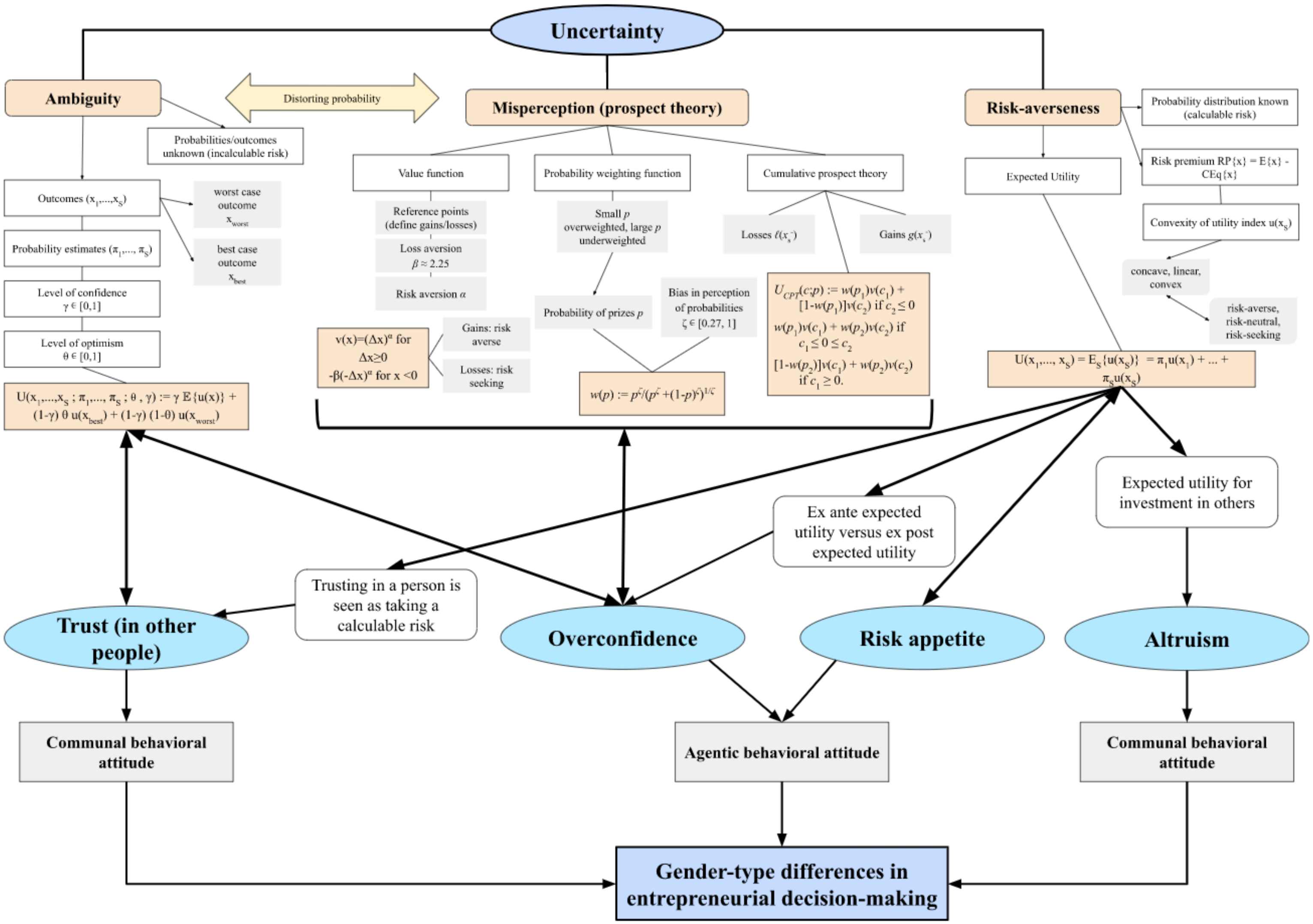
Appendices

Appendix 1: Conceptual framework on uncertainty towards mapping the gender dimension of entrepreneurial decision-making



Source: Authors' illustration.

Appendix 2: Mapping of the theories of uncertainty behind the concepts



Source: Authors’ illustration.

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